BFS

Breadth First Search

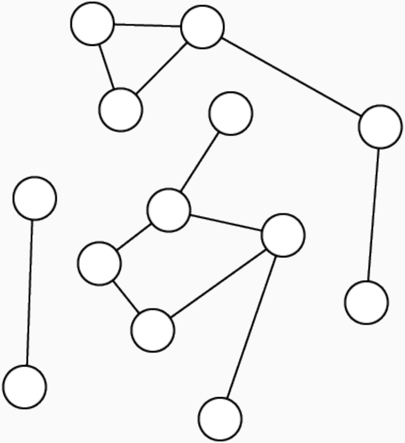
Daniel Graf (Slides by Benjamin Schmid)

2017-11-04

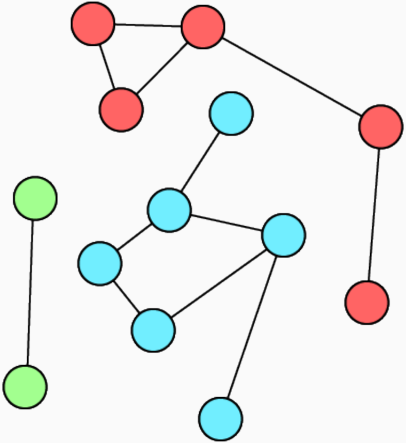
Swiss Olympiad in Informatics

Introduction

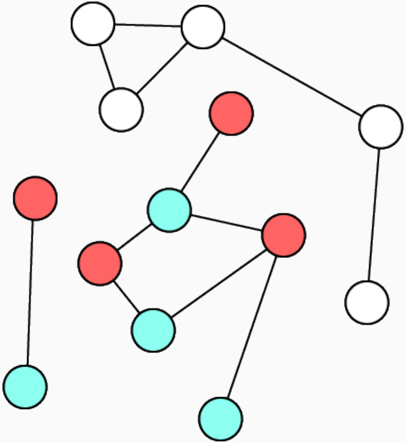
Graph traversal



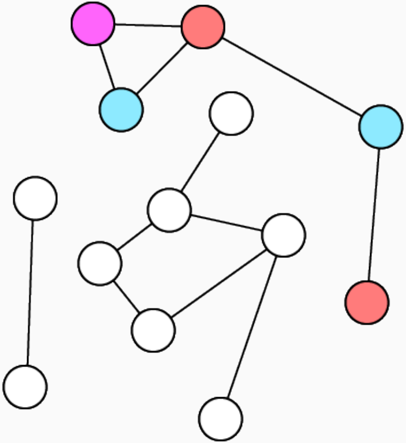
Components



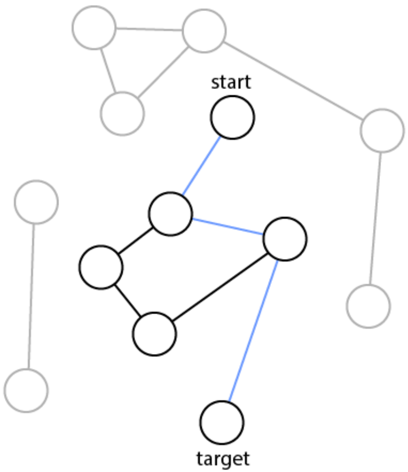
Bipartite



Bipartite

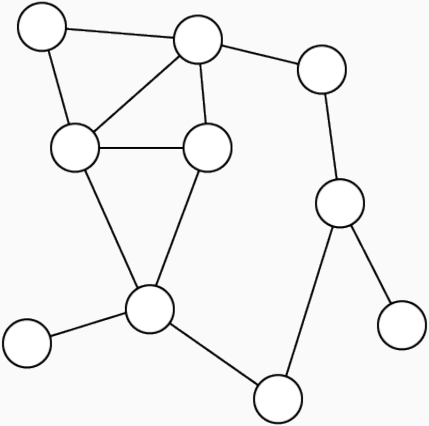


Shortest path

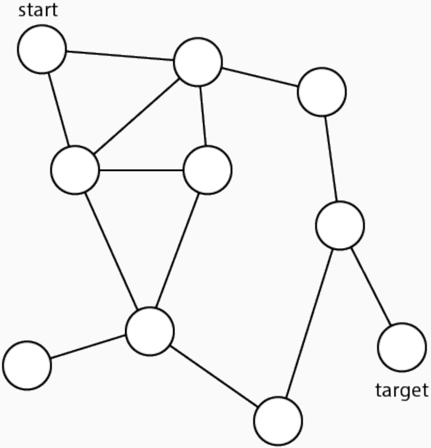


Shortest path

Shortest path



Shortest path



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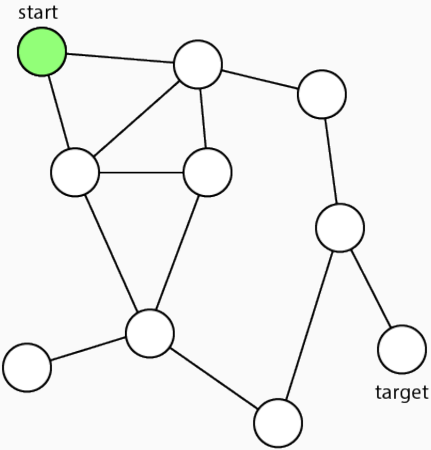

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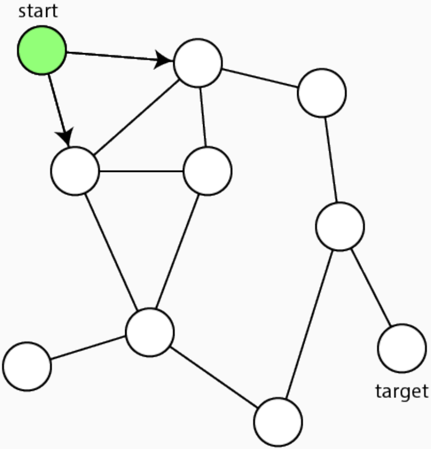

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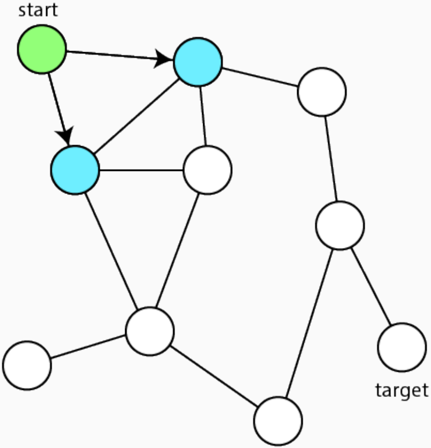
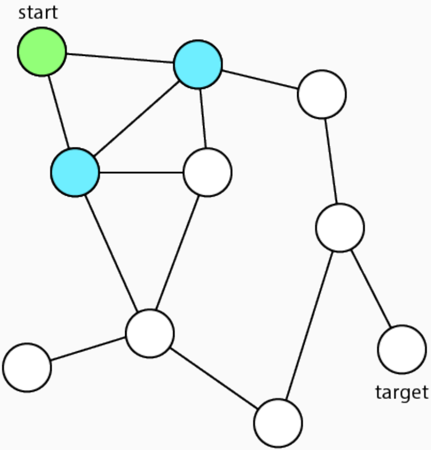
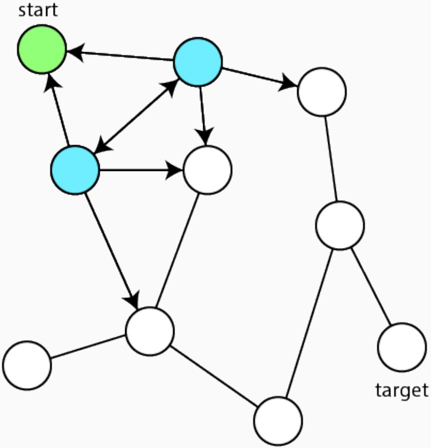


Shortest path



Shortest path



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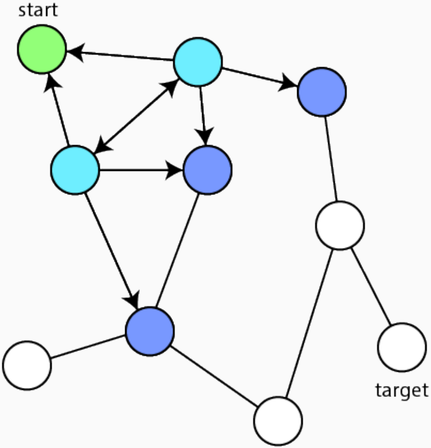
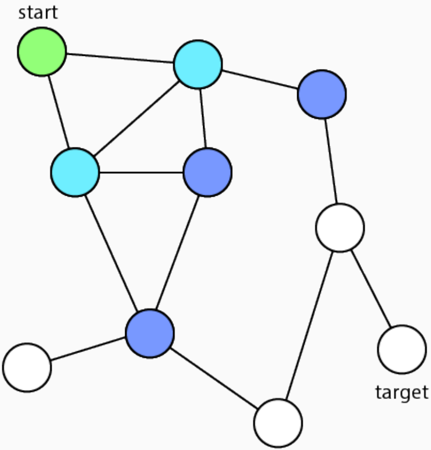
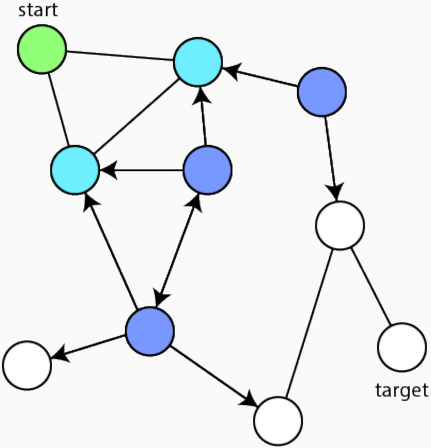


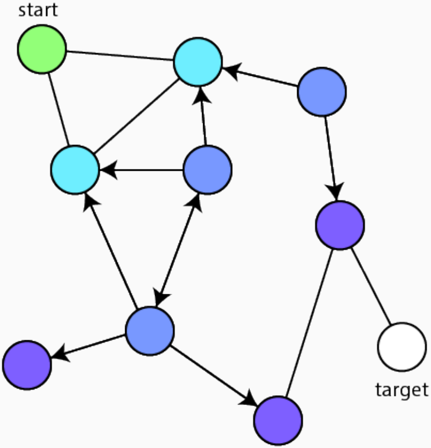
Shortest path



Shortest path



Shortest path



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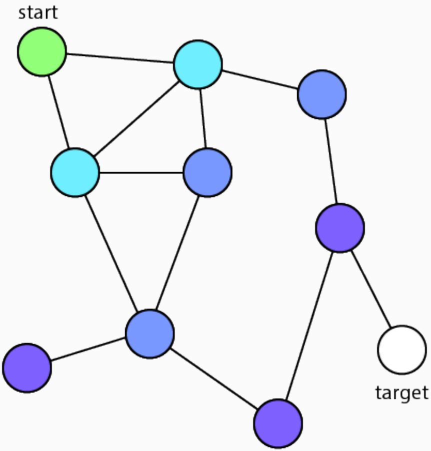

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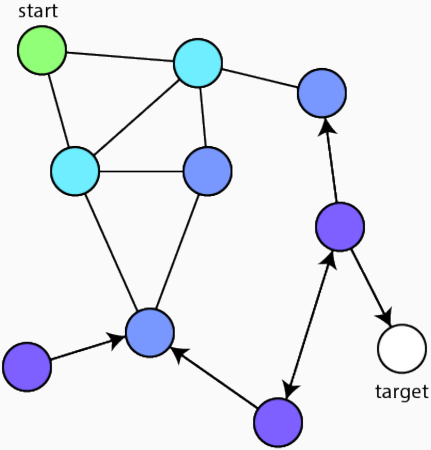
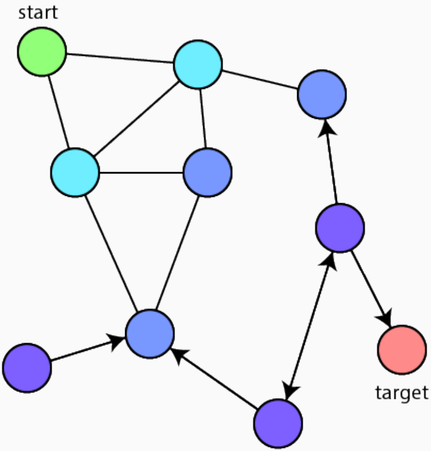
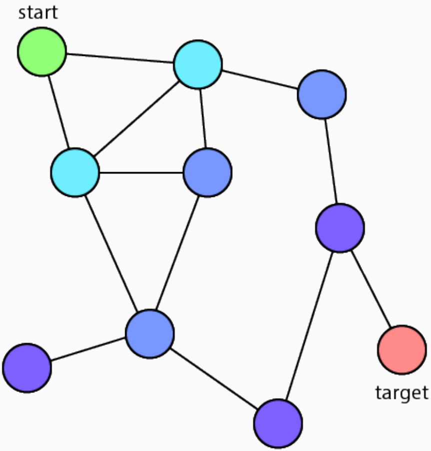


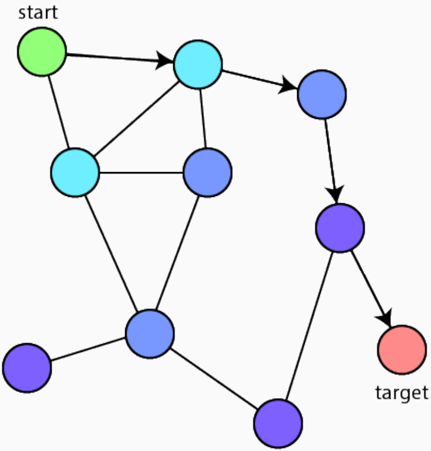
Shortest path



Shortest path



Shortest path

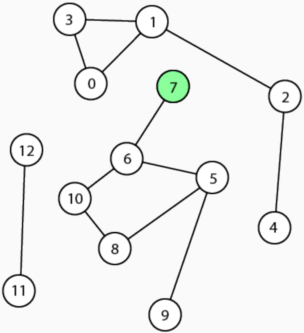


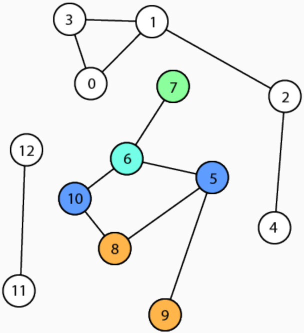
BFS

- Breadth First Search (BFS)
- First explore breadth
- Nodes visited in order of distance to start
- Thus find shortest path (if equal lengths)

BFS Implementation

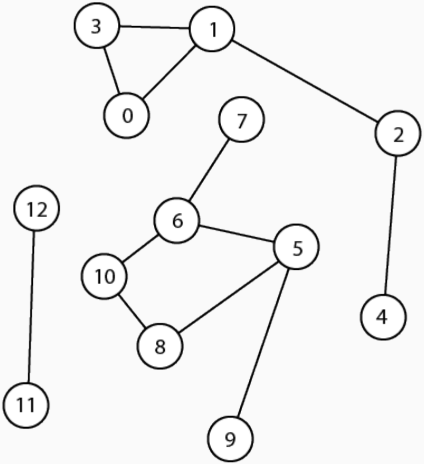
Find Component





Generations / Queue

Generations:
[ ]

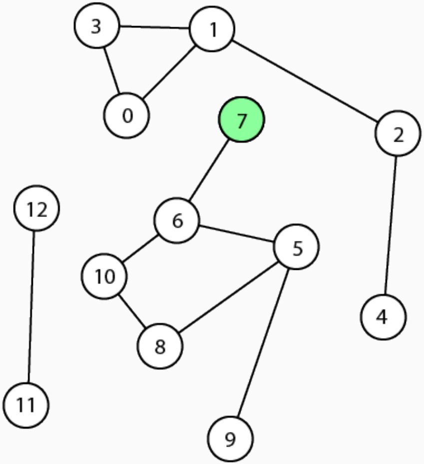


Generations / Queue

Generations:

[7]

[ ]

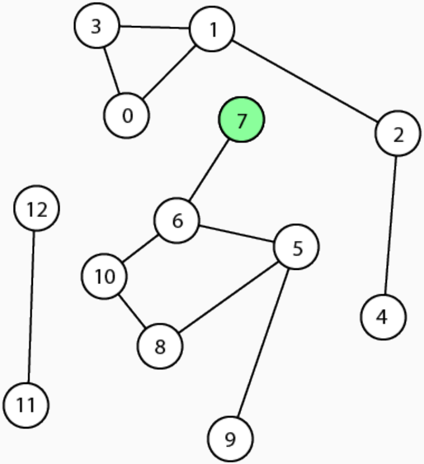


Generations / Queue

Generations:

[7]

[6]



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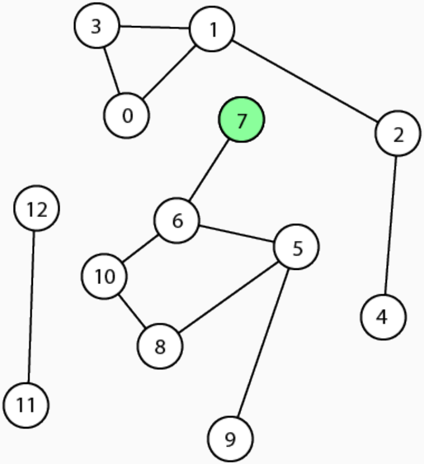

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[6]

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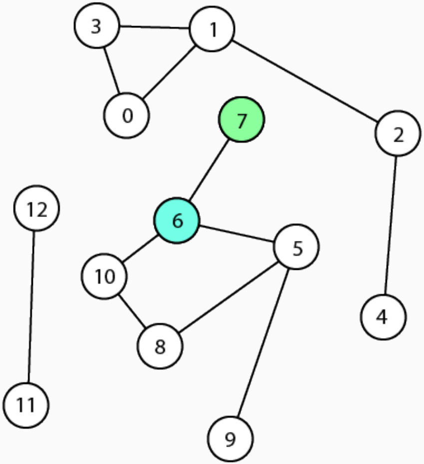


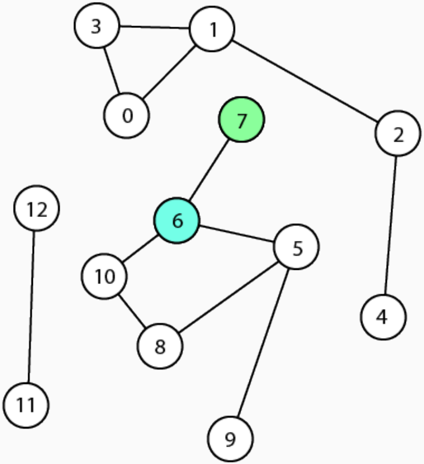
Generations / Queue

Generations:

[7]

[6]

[10, 5]



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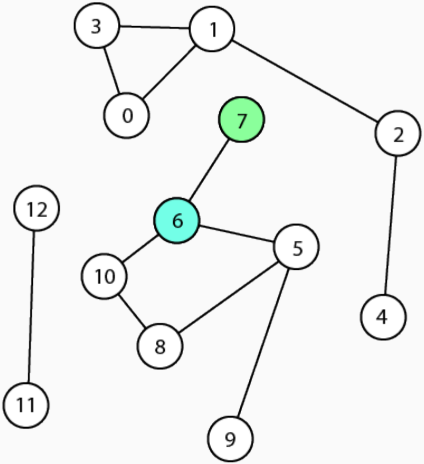

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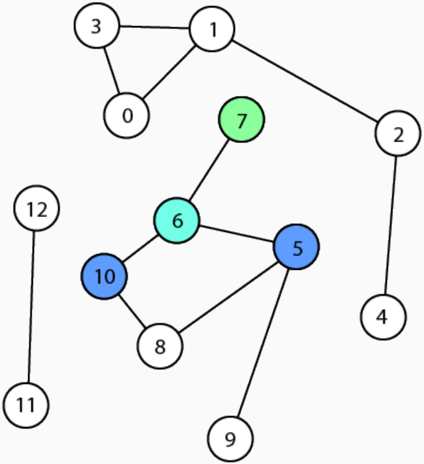
[6]

[10, 5]

[ ]



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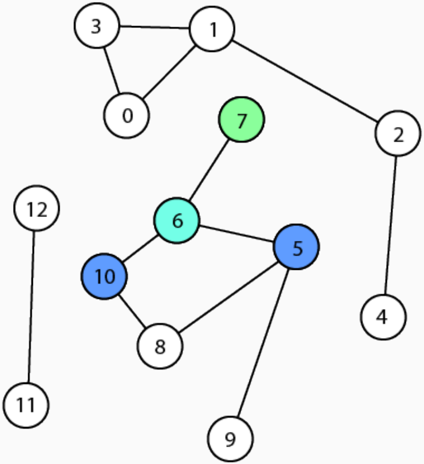
Generations:

[7]

[6]

[10, 5]

[8]



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[10, 5]

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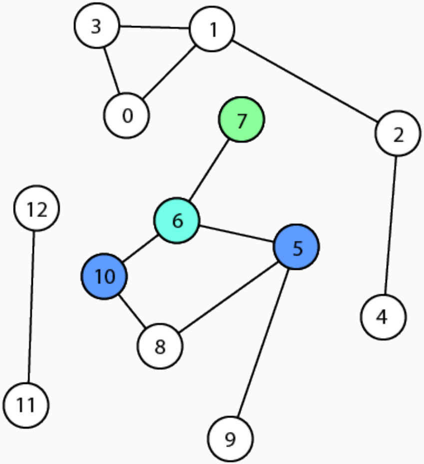

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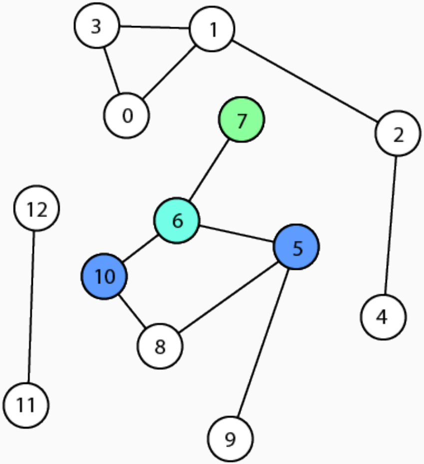
[6]

[10, 5]

[8]



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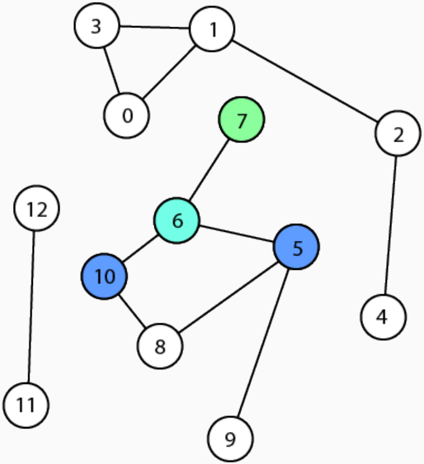
Generations:

[7]

[6]

[10, 5]

[8, 9]



Generations / Queue

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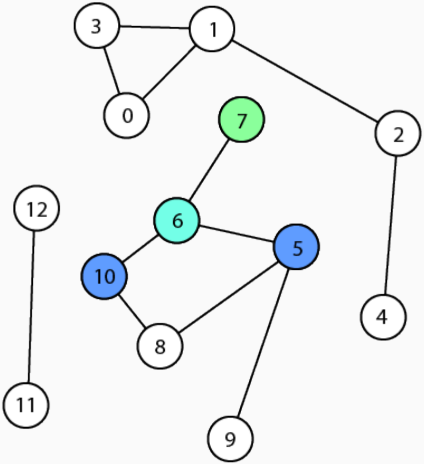
[6]

[10, 5]

[8, 9]



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Generations:

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[6]

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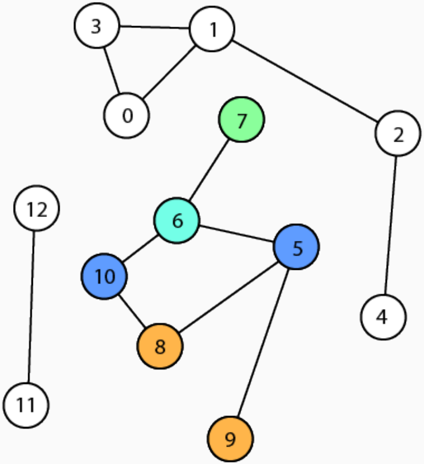
[ ]



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[6]

[10, 5]

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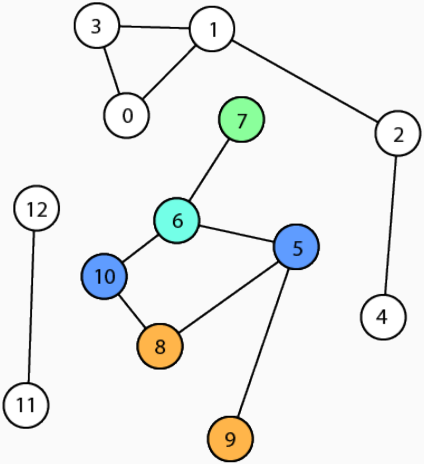

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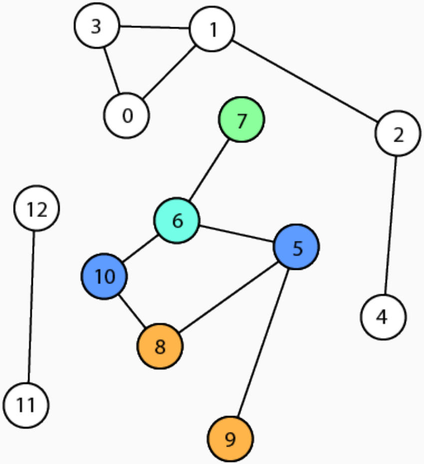

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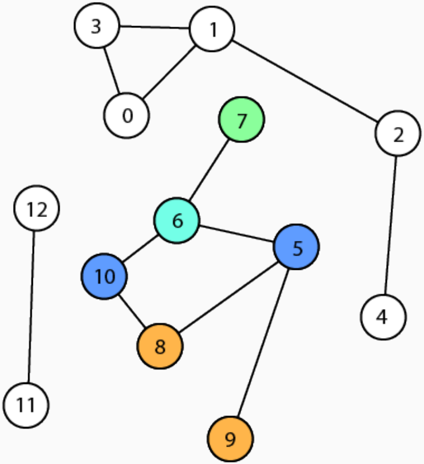


Generations / Queue

Generations

[7]

[6]

[10, 5]

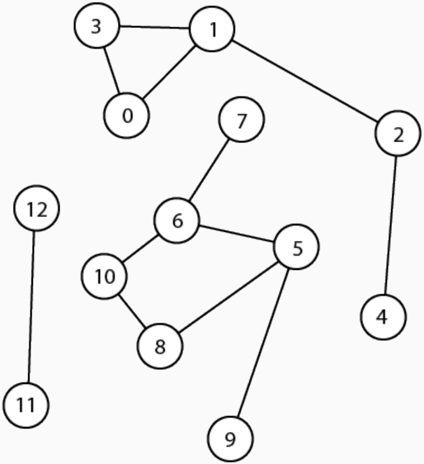
[8, 9]

Queue (first in, first out)

[7, 6, 10, 5, 8, 9]

Adjacency List

0: [1, 3]
1: [0, 2, 3]
2: [1, 4]
3: [0, 1]
4: [2]
5: [6, 8, 9]
6: [5, 7, 10]
7: [6]
8: [5, 10]
9: [5]
10: [6, 8]
11: [12]
12: [11]



BFS Implementation

- Visited flag for each node
- Queue to store neighbors
- `graph` is adjacency list

```
from collections import deque

def bfs(start):
  q = deque()
  visited = [False] * len(graph)
```

BFS Implementation

- Process start node

```
def bfs(start):
    ...
    visited[start] = True
    q.appendleft(start)
```

BFS Implementation

- Process neighbors
- Check whether not visited
- Add to queue and set visited

```
def bfs(start):
    ...
    while len(q) > 0:
        current = q.pop()
        for neighbor in graph[current]:
            if not visited[neighbor]:
                q.appendleft(neighbor)
                visited[neighbor] = True
```

BFS Implementation

- Return visited nodes

```python
def bfs(start, end):
    ...
    return visited
```

BFS Implementation

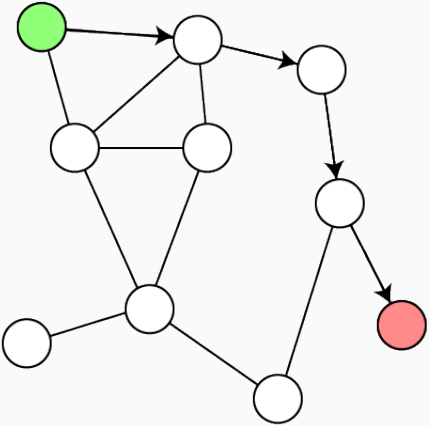
```python
def bfs(start):
    q = deque() # initialize
    visited = [False] * len(graph)

    visited[start] = True
    q.appendleft(start)

    while len(q) > 0: # traverse graph
        current = q.pop()
        for neighbor in graph[current]:
            if not visited[neighbor]:
                q.appendleft(neighbor)
                visited[neighbor] = True
    return visited
```

Shortest Distance Implementation

Shortest Distance



Shortest Distance Implementation

1. List of distance to start
2. Upon adding to queue, store distance

Shortest Distance Implementation

- List of distance to start

```
def bfs(start, target):
    ...
    visited = [False] * len(graph)
    distance = [0] * len(graph)
    ...
```

Shortest Distance Implementation

- Store distance

```
def bfs(start, target):
    ...
    visited[neighbor] = True
    distance[neighbor] = distance[current] + 1
    ...
```

Shortest Distance Implementation

- Return shortest distance
- Note: we know shortest distance to every node

```python
def bfs(start, target):
    ...
    return distance[target]
```

Shortest Distance Implementation

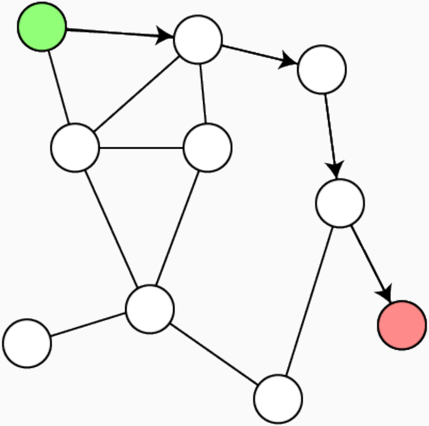
```python
def bfs(start, target):
  q = deque() # initialize
  visited = [False] * len(graph)
  distance = [0] * len(graph)

  visited[start] = True
  q.appendleft(start)

  while len(q) > 0: # traverse graph
    current = q.pop()
    for neighbor in graph[current]:
      if not visited[neighbor]:
        q.appendleft(neighbor)
        visited[neighbor] = True
        distance[neighbor] = distance[current] + 1
  return distance[target]
```

Shortest Path Implementation

Shortest Path



Shortest Path Implementation

1. Store ”parent” of node
2. Upon adding to queue, store parent

Shortest Path Implementation

- List of parents

```
def bfs(start, target):
    ...
    visited = [False] * len(graph)
    parent = [-1] * len(graph)
    ...
```

Shortest Path Implementation

- Store parent

```
def bfs(start, target):
    ...
    visited[neighbor] = True
    parent[neighbor] = current
    ...
```

Shortest Path Implementation

- Return shortest path
- Note: we know shortest path to every node

```python
def bfs(start, target):
    ...
    if parent[target] == -1:
        return []
    path = []
    current = target
    while current != -1:
        path.append(current)
        current = parent[current]
    return reversed(path)
```

Shortest Path Implementation

```python
def bfs(start, target):
  q = deque() # initialize
  visited = [False] * len(graph)
  parent = [-1] * len(graph)

  visited[start] = True
  q.appendleft(start)

  while len(q) > 0: # traverse graph
    current = q.pop()
    for neighbor in graph[current]:
      if not visited[neighbor]:
        q.appendleft(neighbor)
        visited[neighbor] = True
        parent[neighbor] = current

  if parent[target] == -1: # reconstruct path
    return []
  path = []
  current = target
  while current != -1:
    path.append(current)
    current = parent[current]
  return list(reversed(path))
```

Runtime

Runtime

- Each node is visited exactly once
- Each edge is visited exactly twice

Runtime

$$\mathcal{O}(n + m)$$

Summary

Advanced

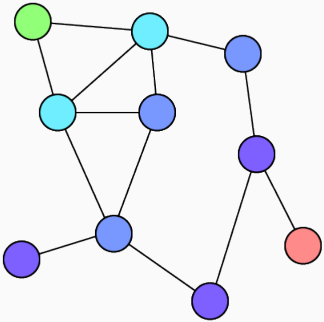
- Works with directed graphs
- Can find shortest path to all nodes
- Sometimes only implicit state

Summary

- Similar to DFS
- Progress "by generation"
- Useful for many different problems
- E.g. components, shortest path, bipartite

Conclusion

Bug Bounty: 1 Prügeli
(expires Sunday 10pm)



And now is your last chance for Pizza!