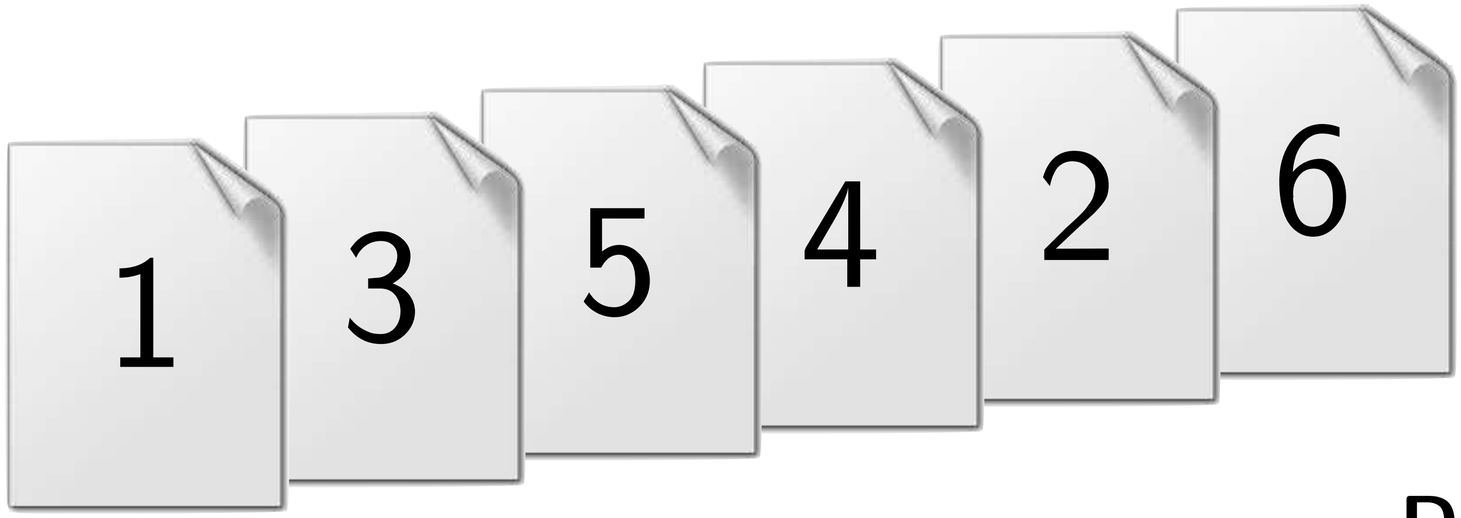
Task „Stofl's Dissertation“

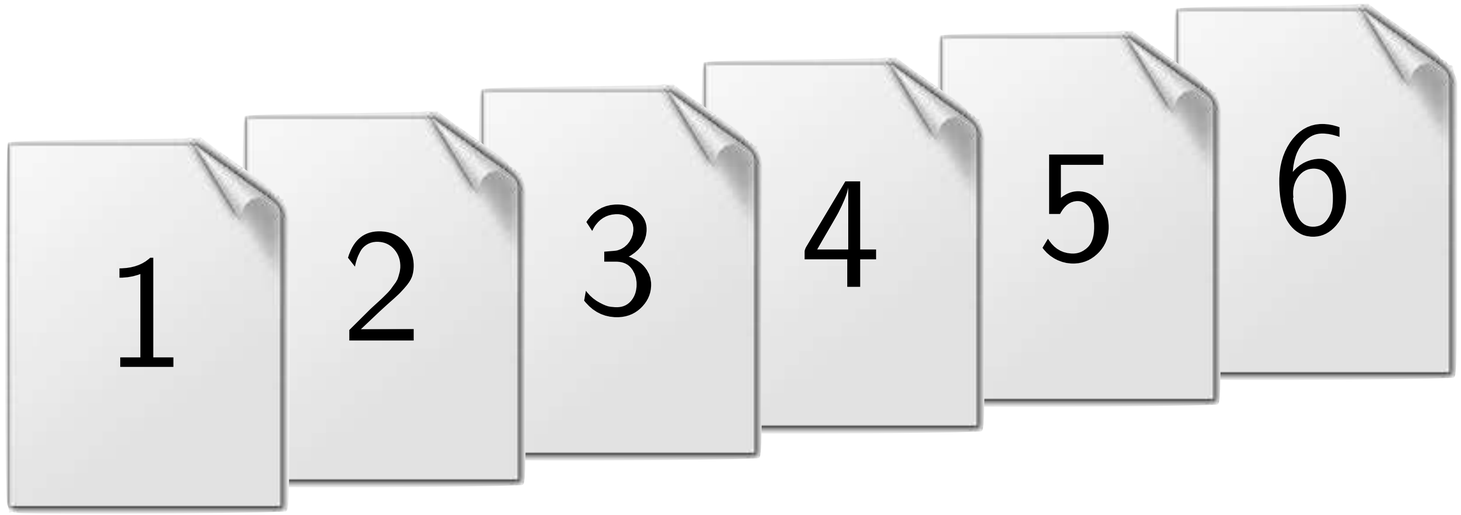
First Round, SOI 2017



Daniel Graf

daniel@soi.ch

Story



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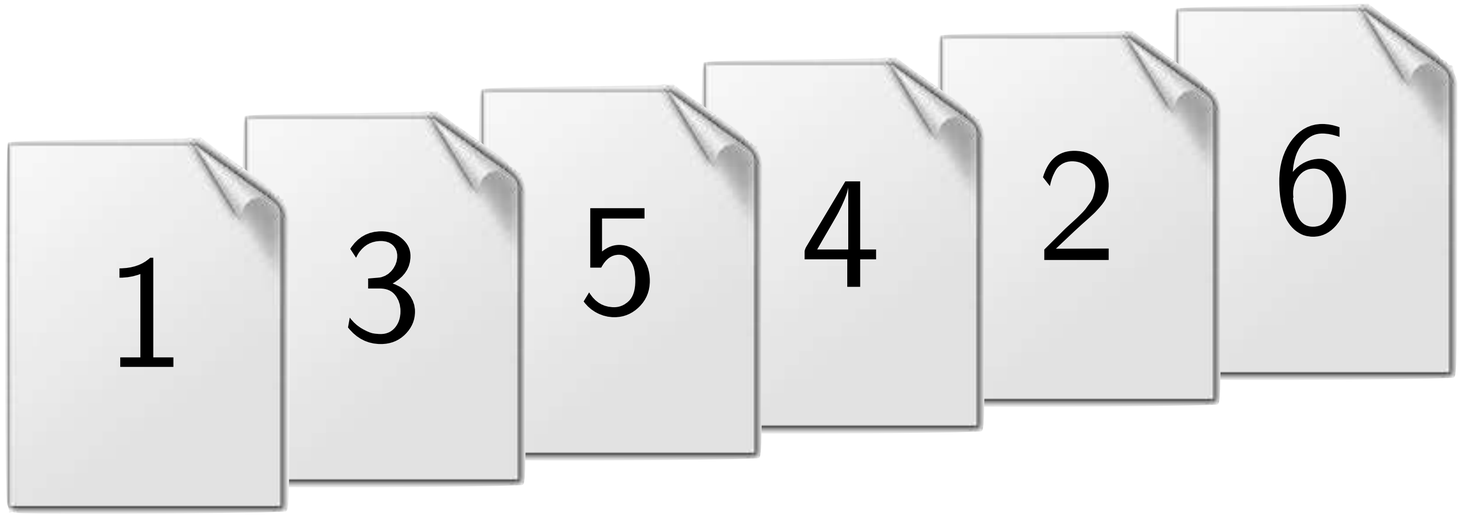
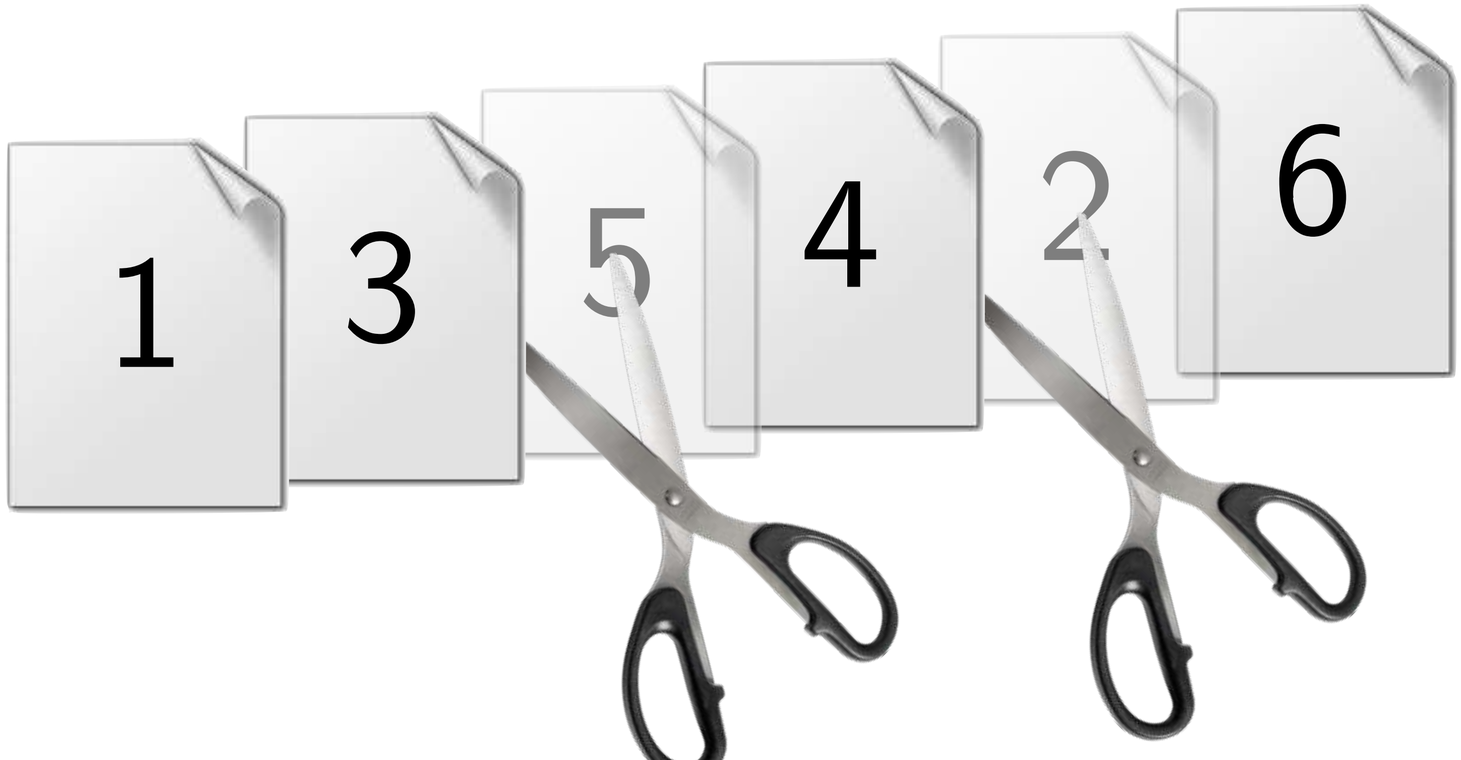
Problem: The pages of the thesis got mixed up.



Story

Problem: The pages of the thesis got mixed up.

Task: Tear out pages so that rest is ascending and as long as possible.



Subtask 1 / Is there a 2-page thesis? / SOI 2017

Input: N the number of pages

$a_1, a_2, a_3, \ldots, a_N$ the page numbers in the printout

Output: *YES* there is at least a thesis of 2 pages

NO all valid theses are only 1 page

Limits: $N \leq 1'000$

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Examples

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5 1 3 2 4	2 1 4 3	3 2 1

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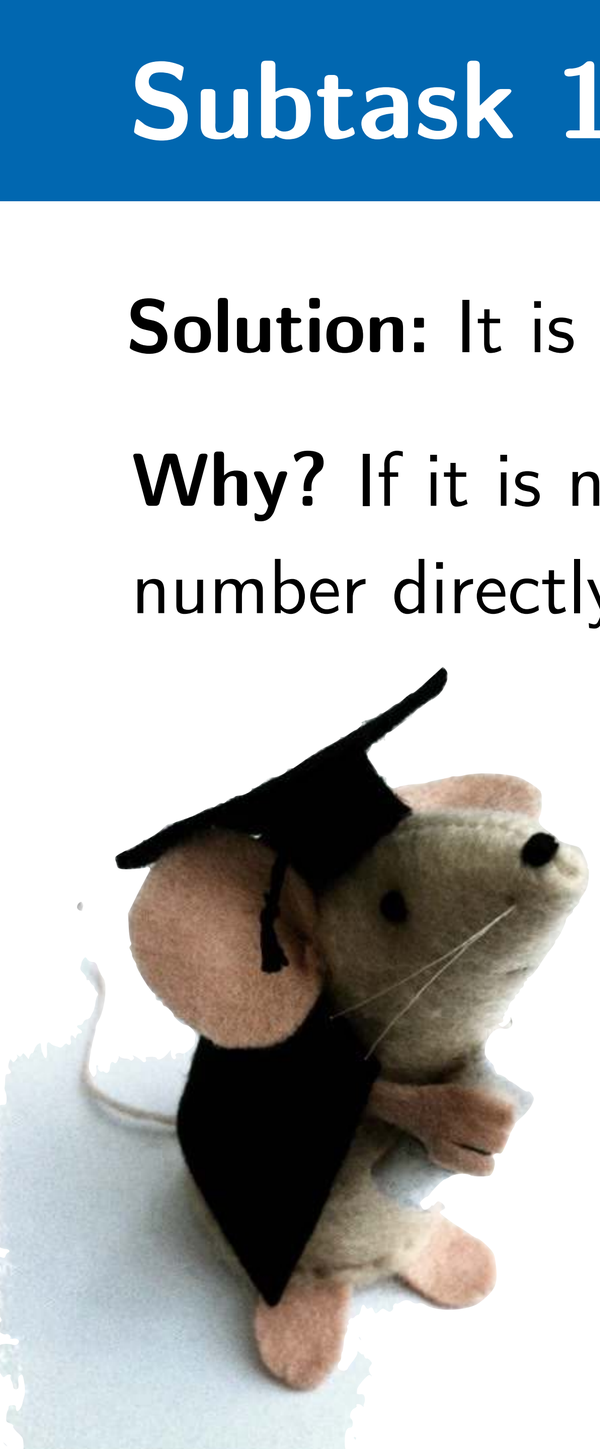

Solution: It is possible unless it is sorted decreasingly.

Why? If it is not always decreasing, there is a smaller page number directly in front of a larger one. We can keep those two.

$N = 6$

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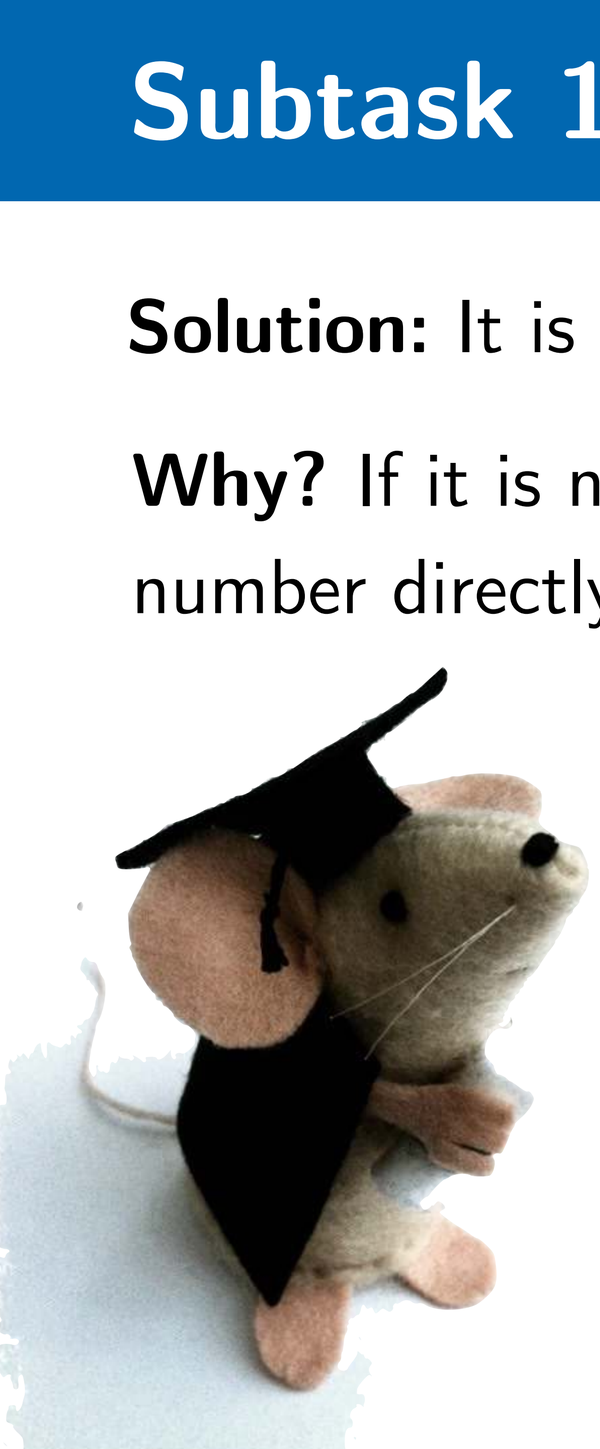

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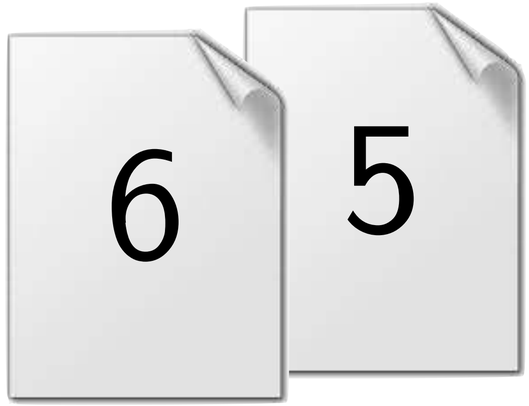

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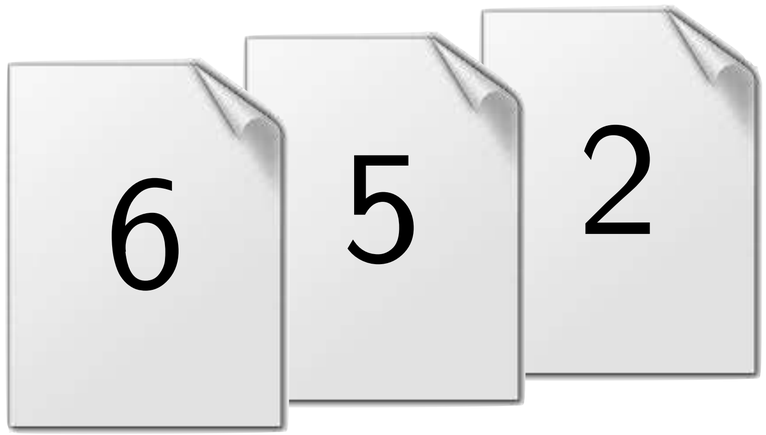

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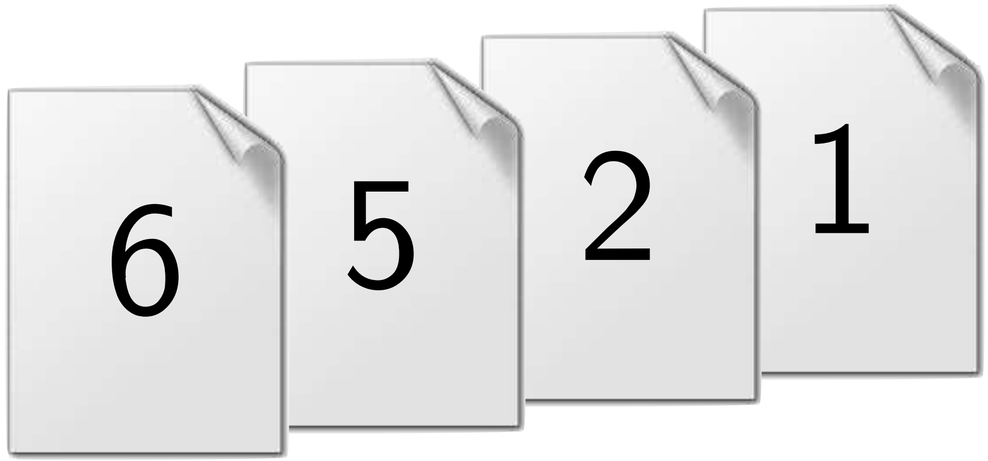

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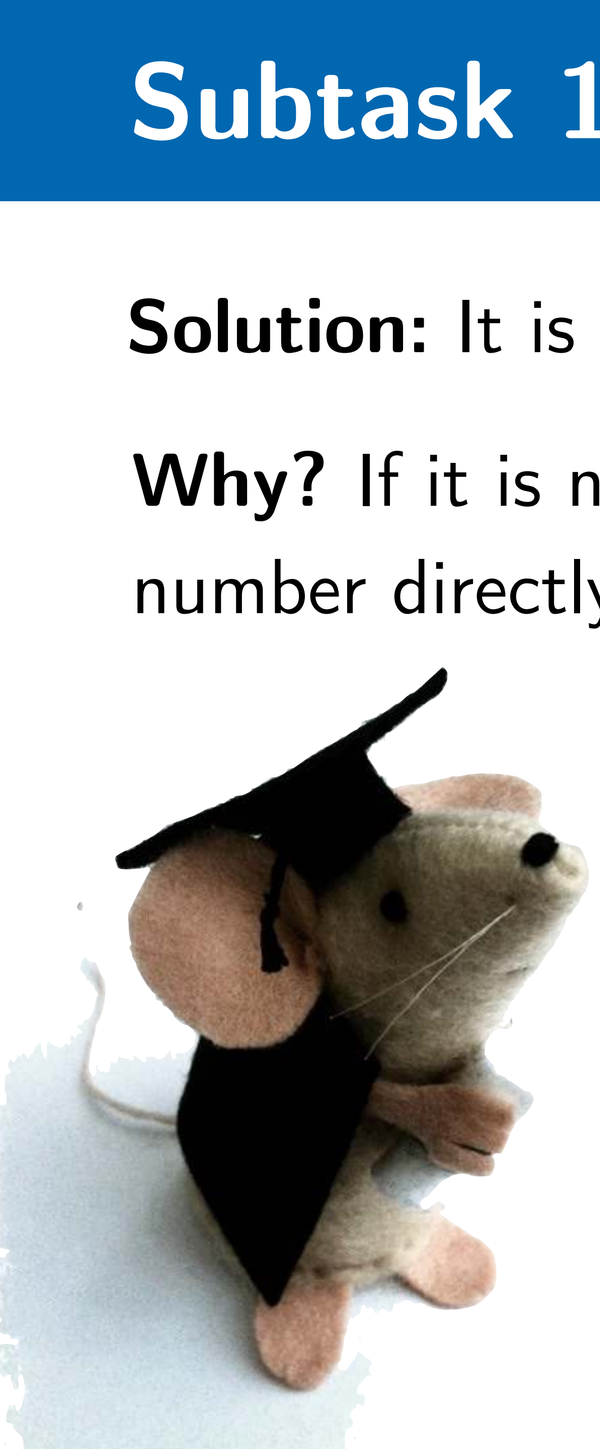

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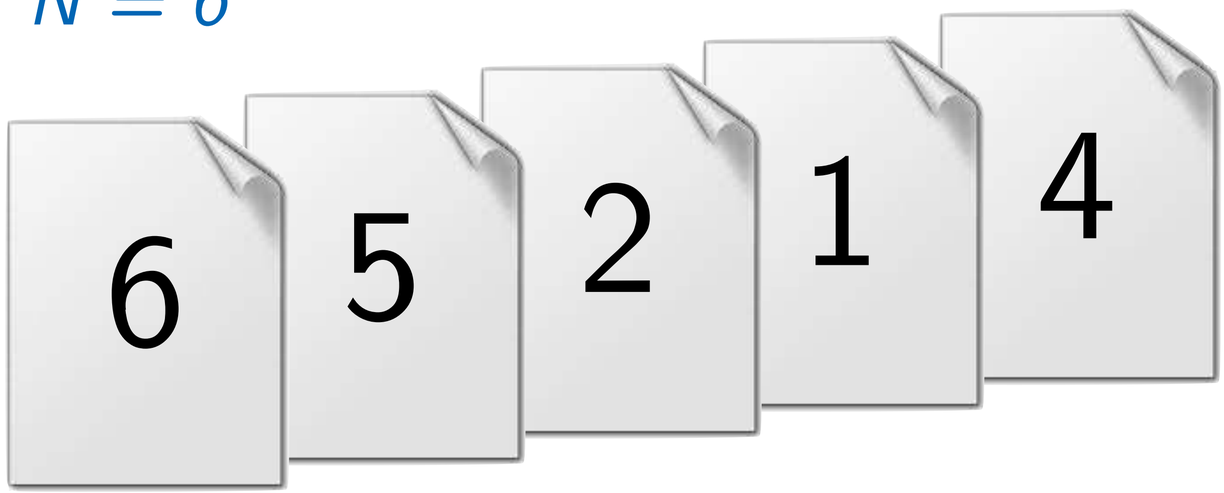

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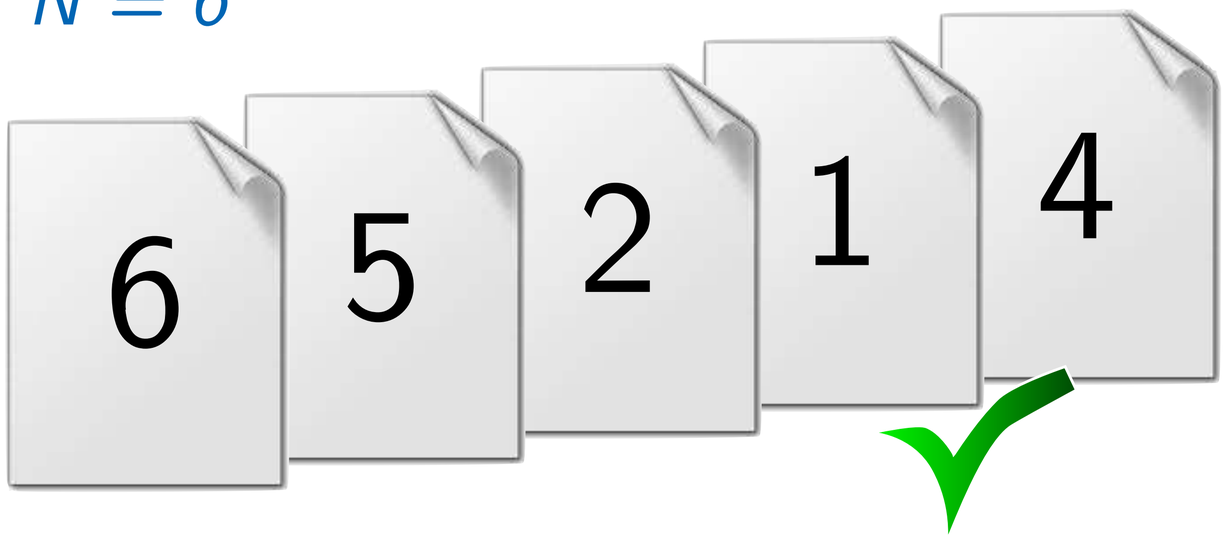

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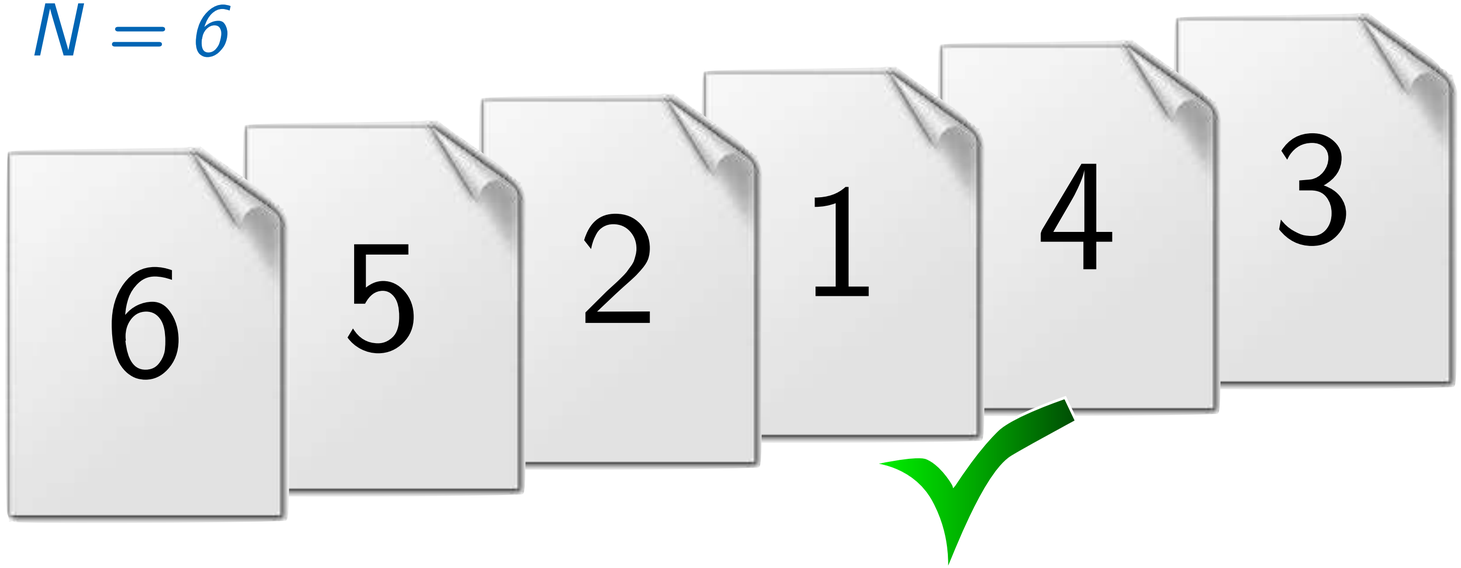

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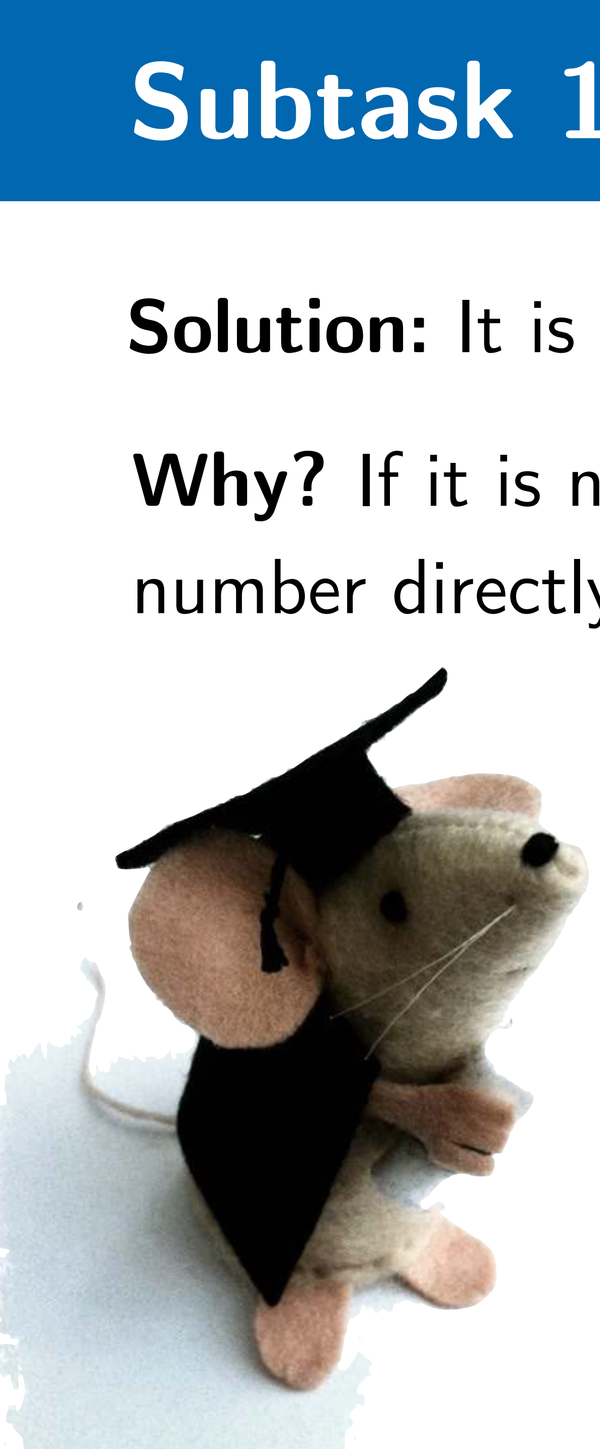

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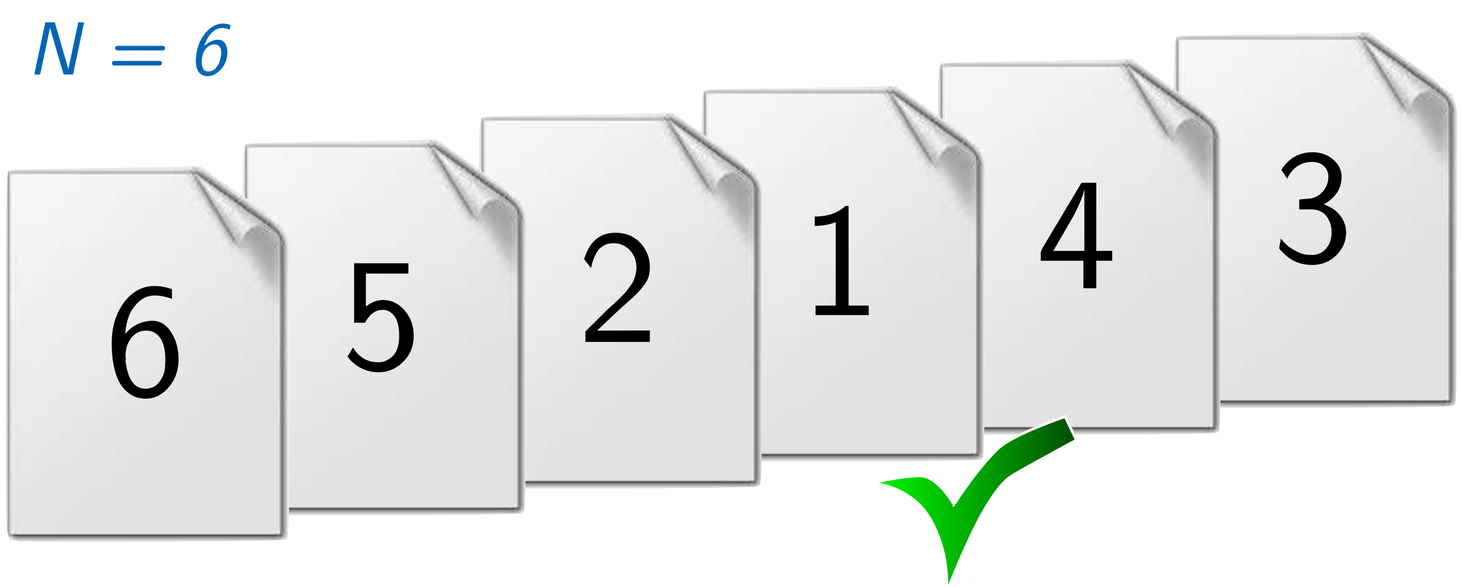

Running time: $O(N)$

10 points

Subtask 2 / How long can it be?

Input: N the number of pages

$a_1, a_2, a_3, \ldots, a_N$ the page numbers in the printout

Output: M length of the longest possible thesis

Limits: $N \leq 10'000$

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Examples		
3 2 1	2 1 4 3	5 1 3 2 4

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Examples		
3 2 1	2 1 4 3	5 1 3 2 4
M = 1	M = 2	M = 3

Subtask 2 / How long can it be?

Solution? Trying out all 2^N subsequences is way too slow.

Idea: Go through the pages one by one.

Add the new page to the longest thesis that ends with a smaller page.

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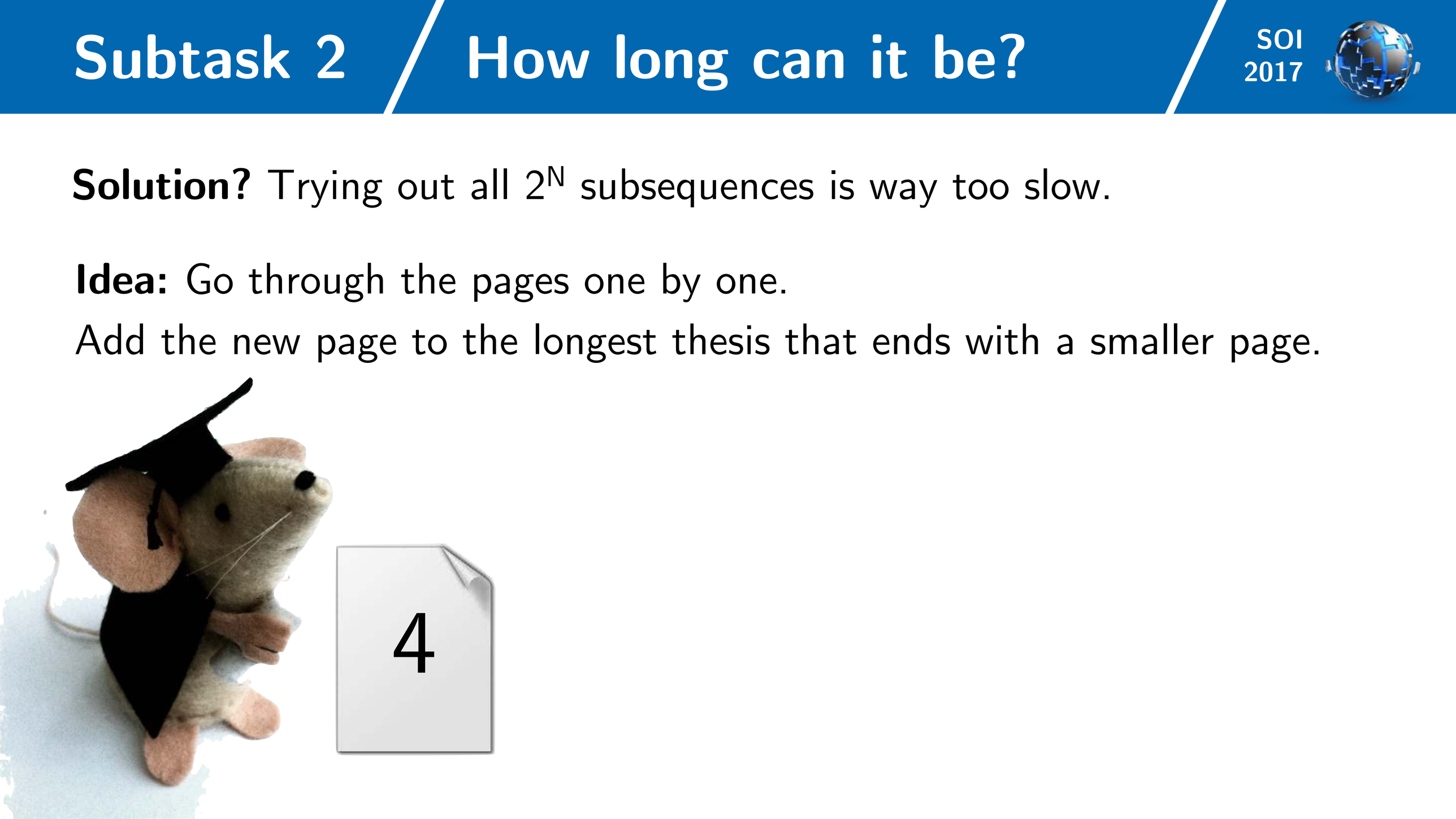

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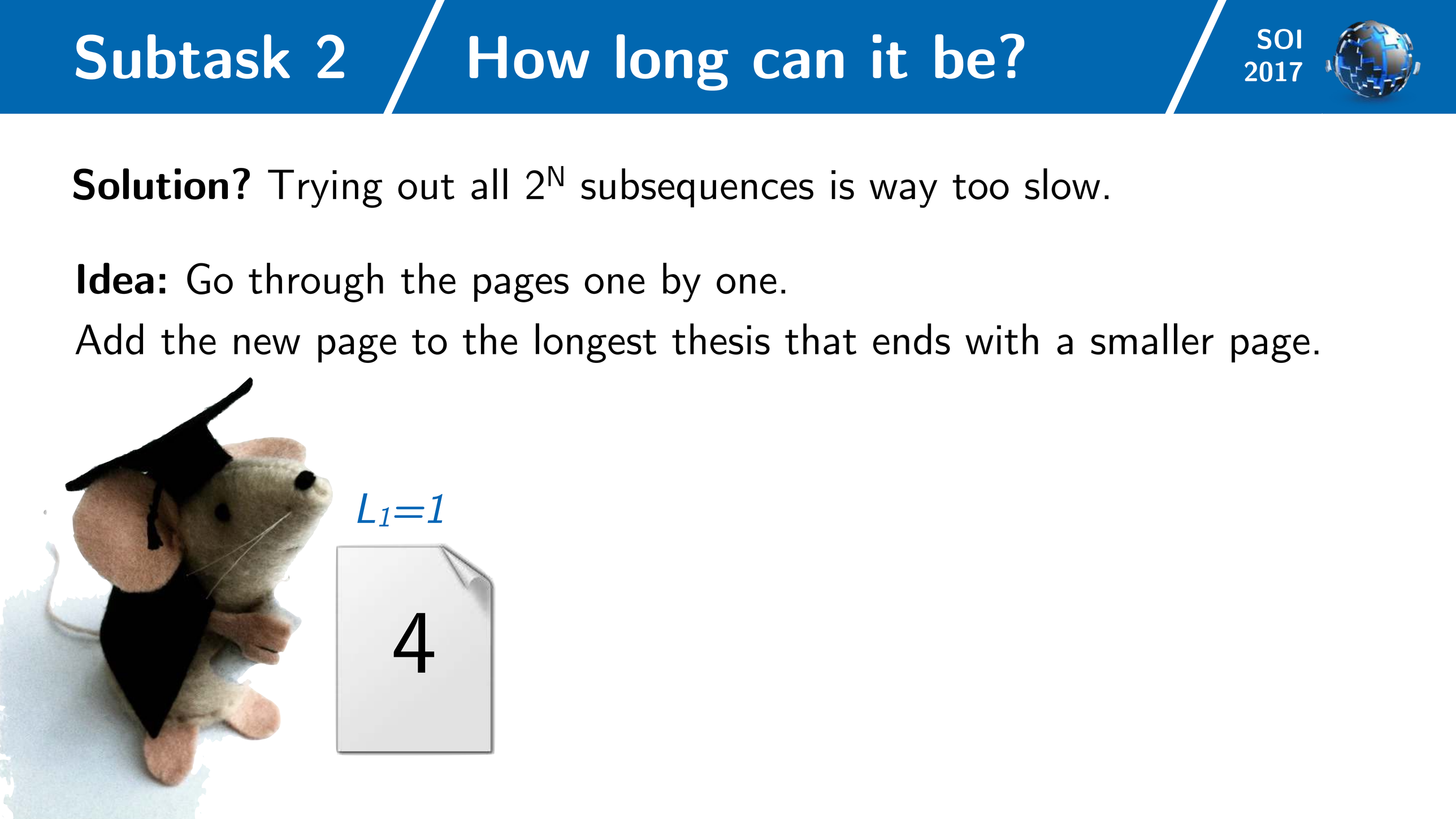

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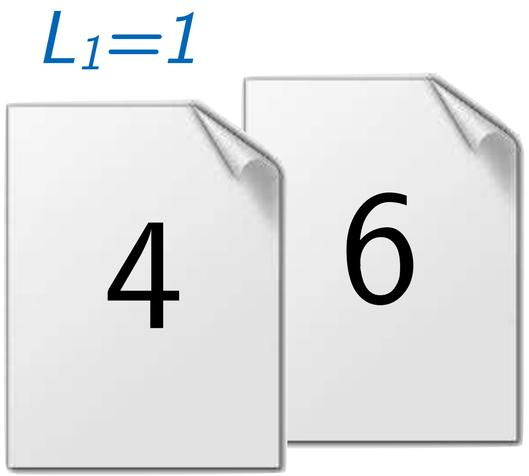

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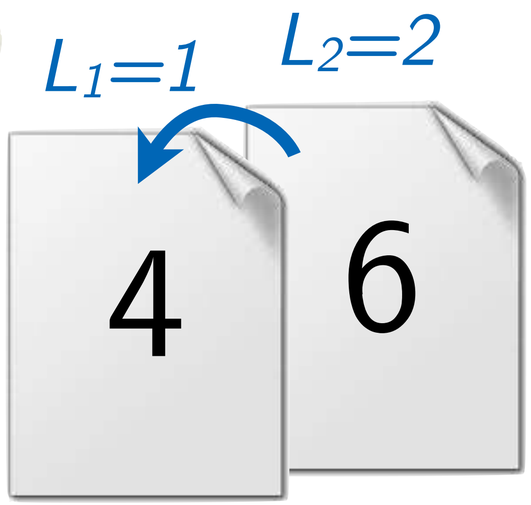

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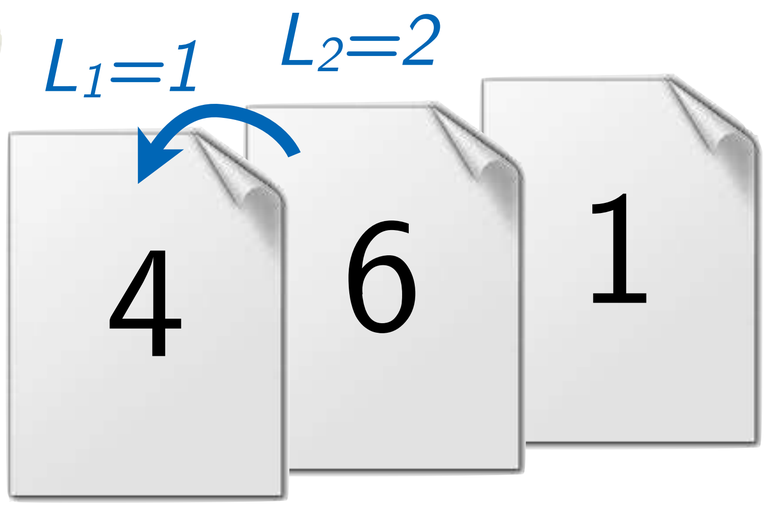

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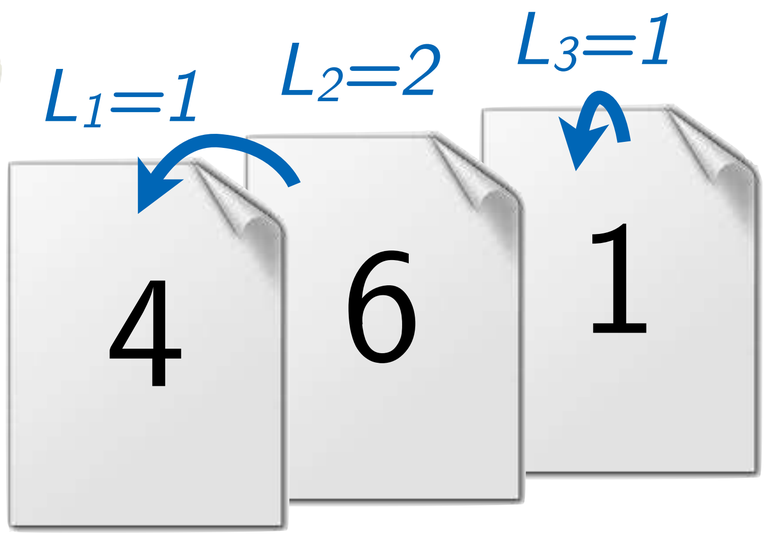

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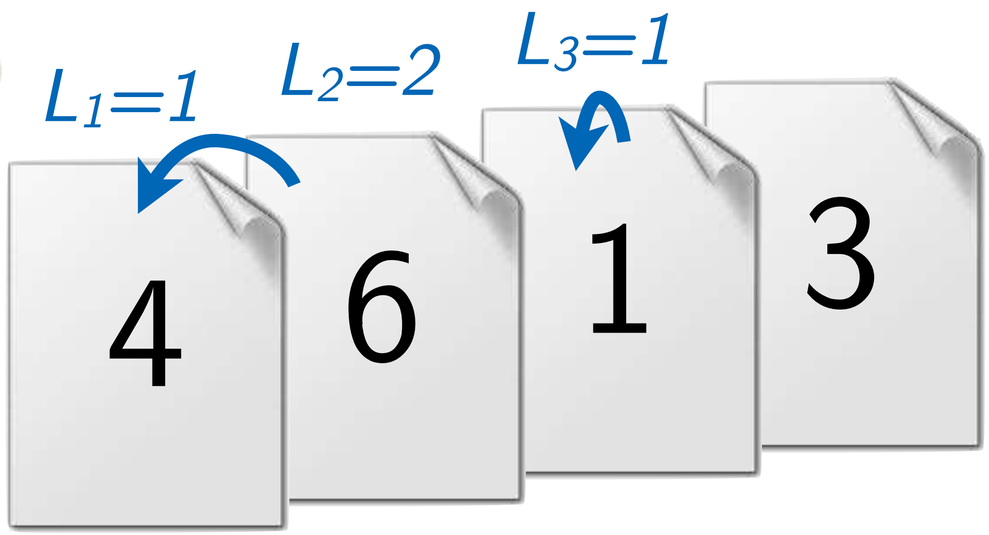

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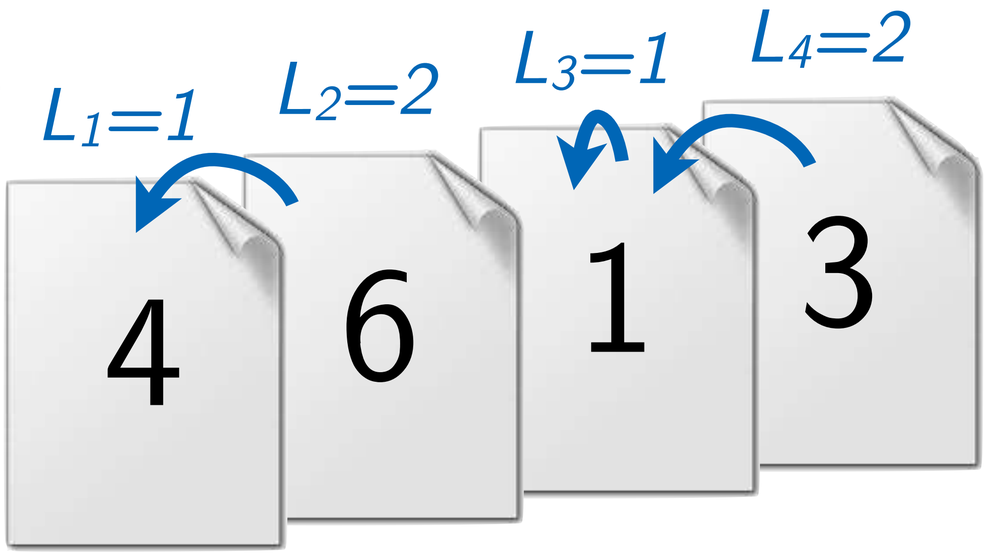

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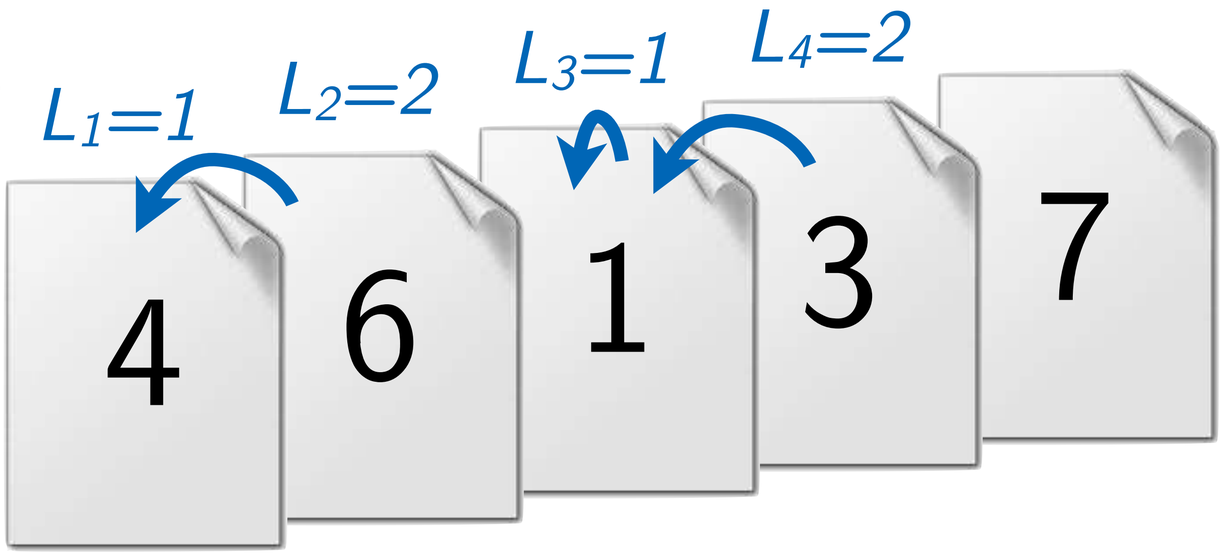

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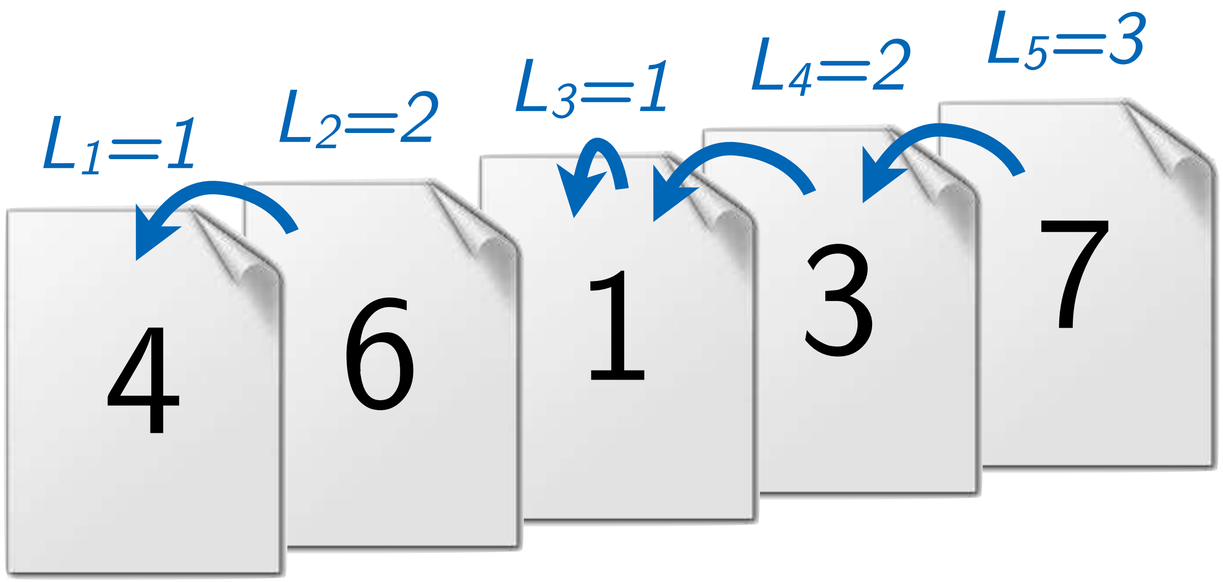

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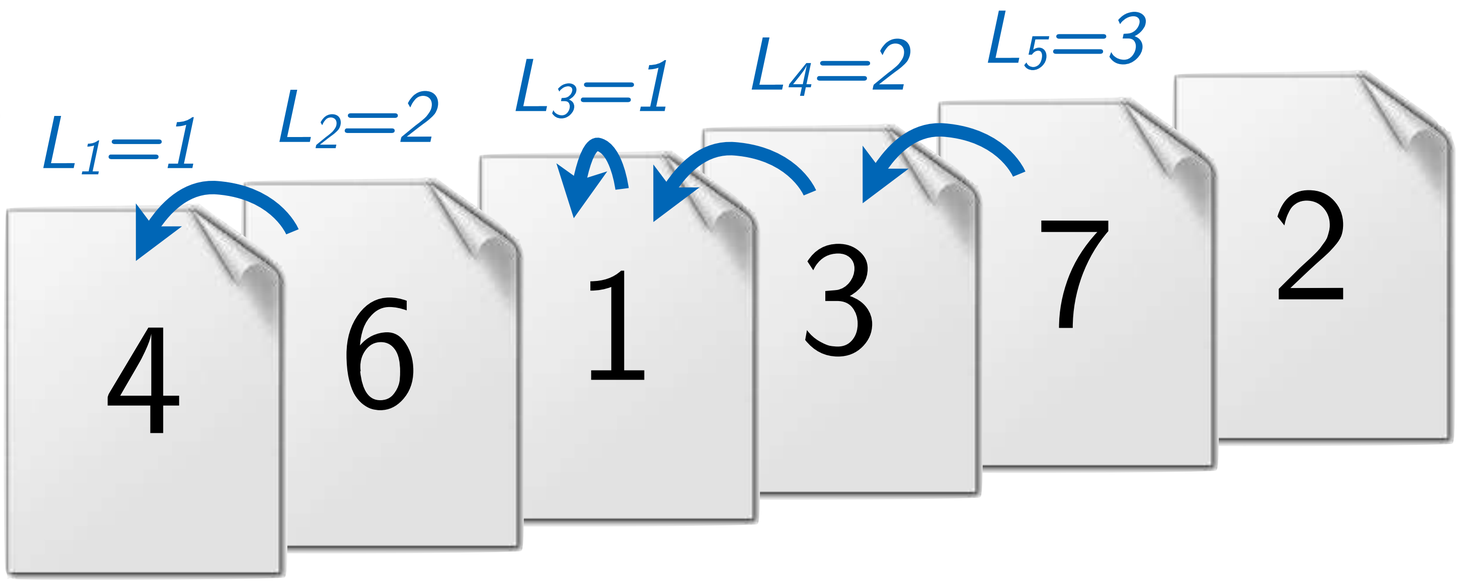

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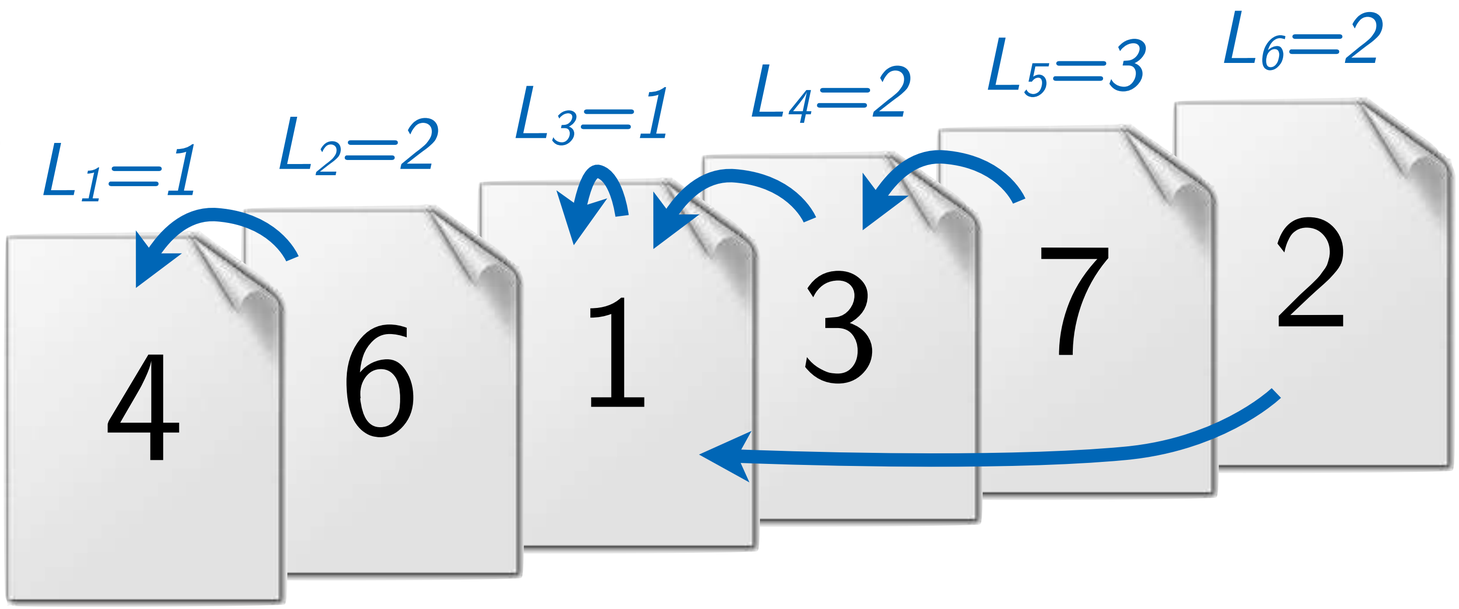

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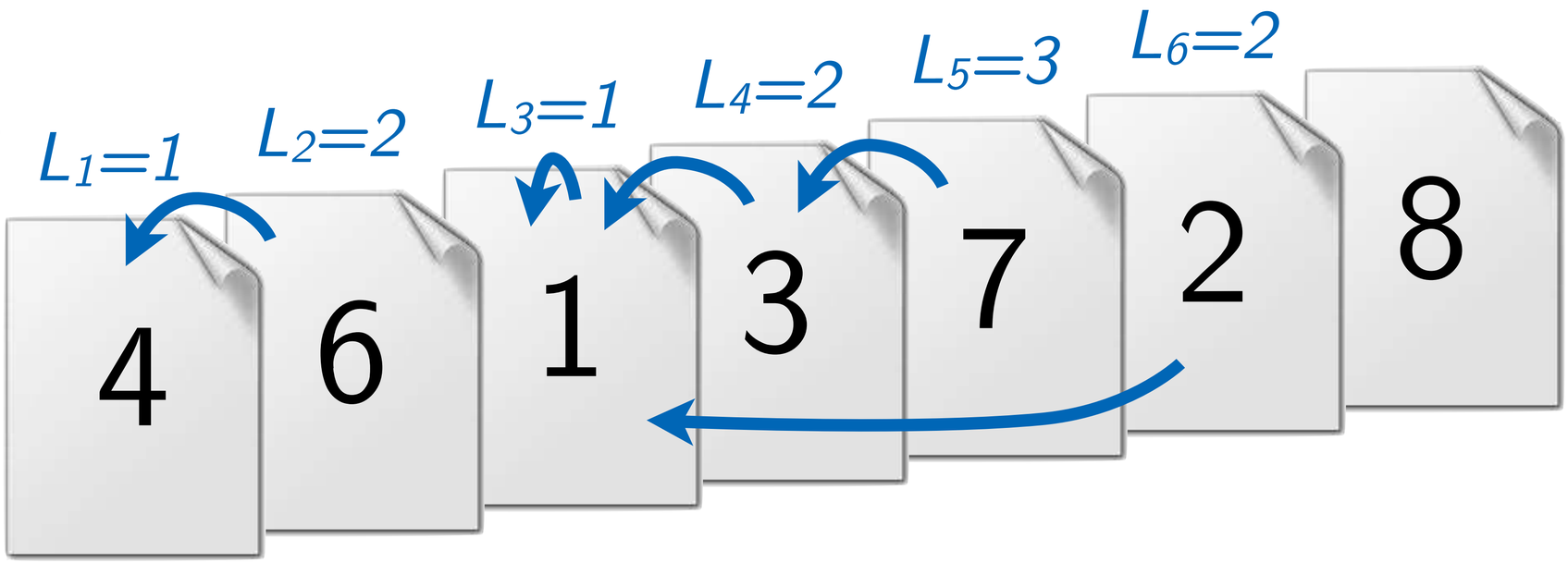

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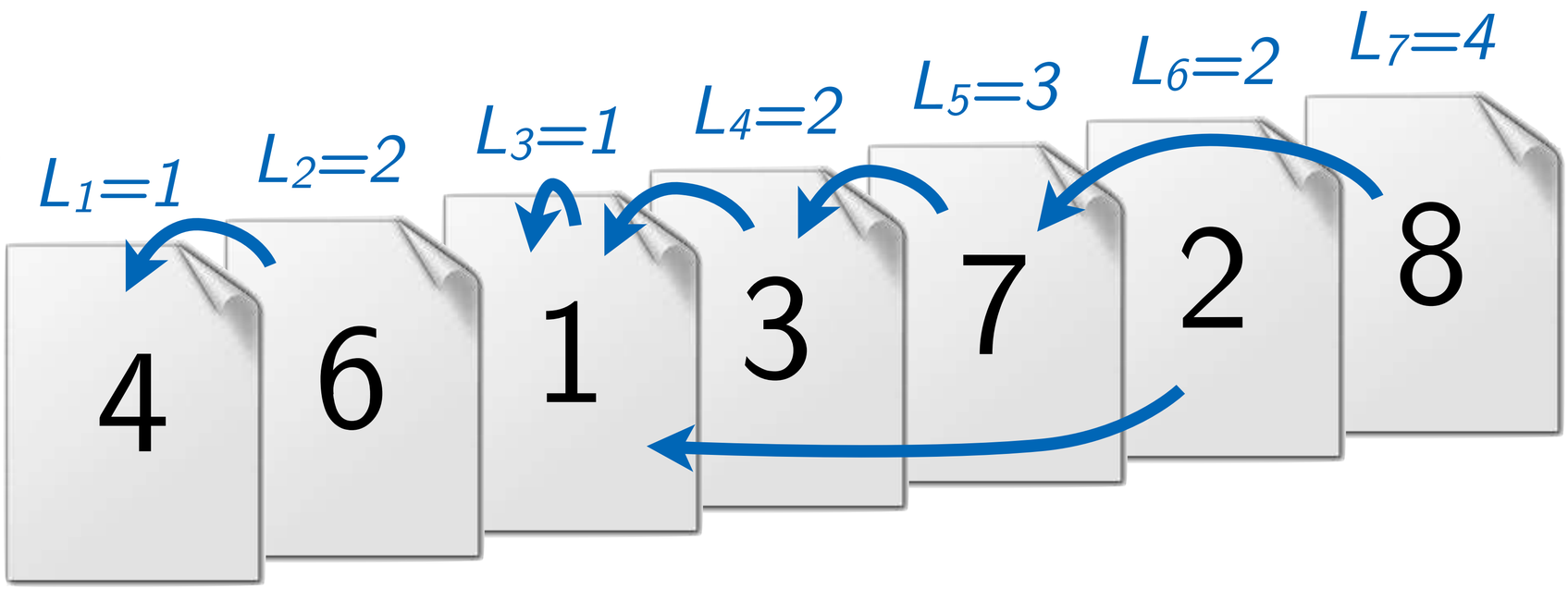

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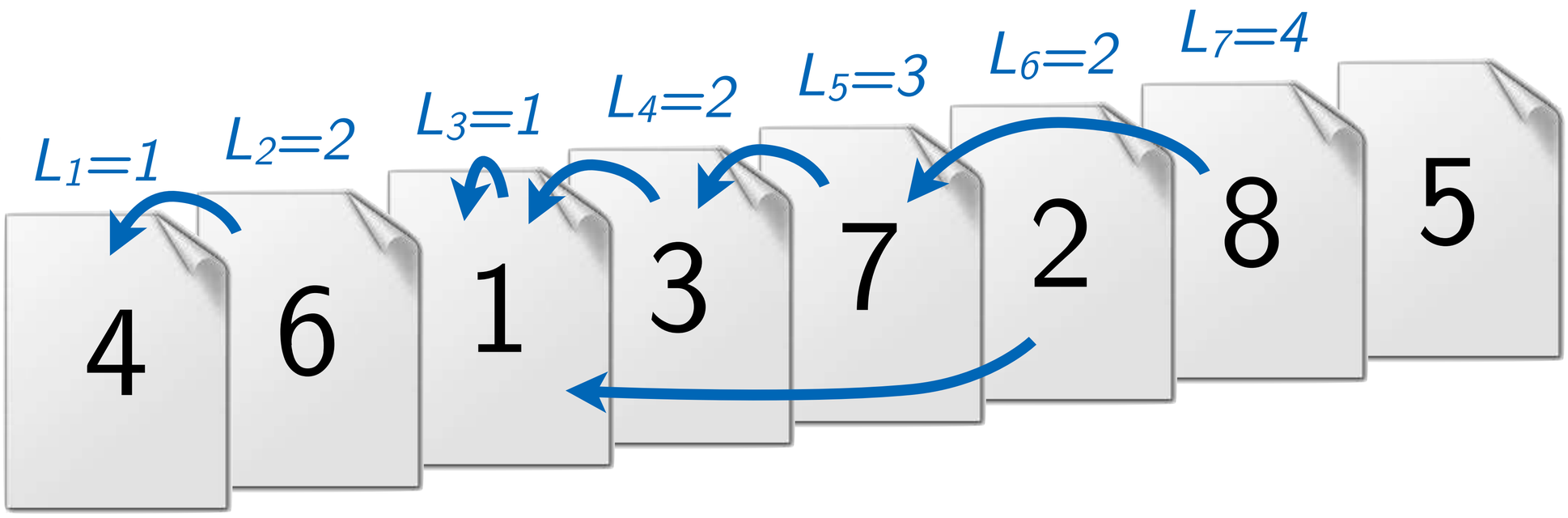

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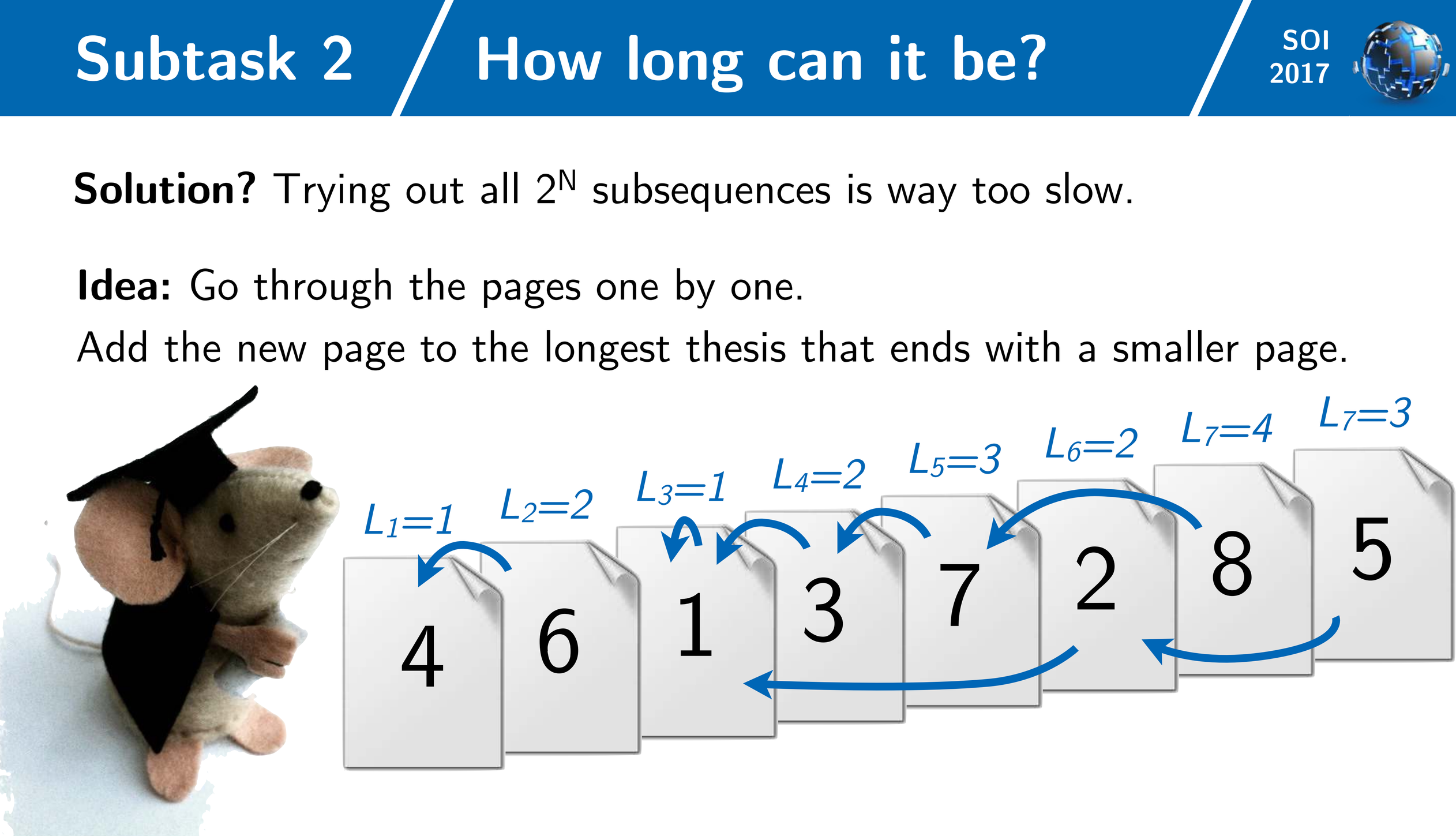

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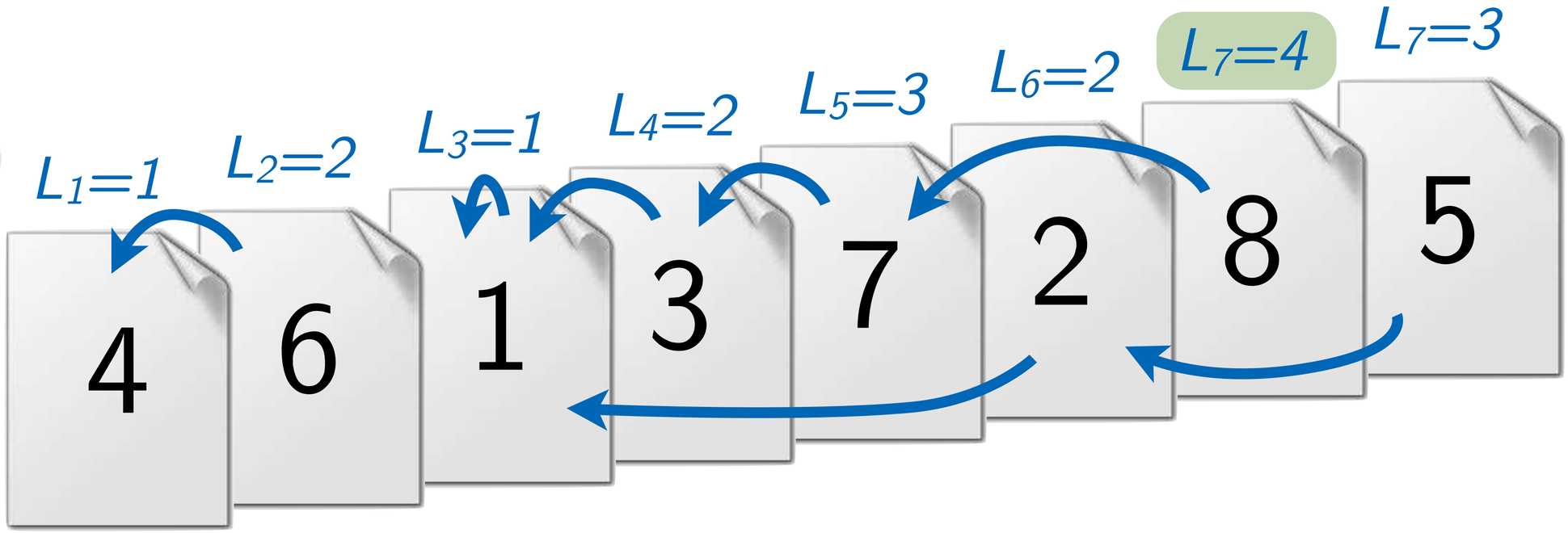

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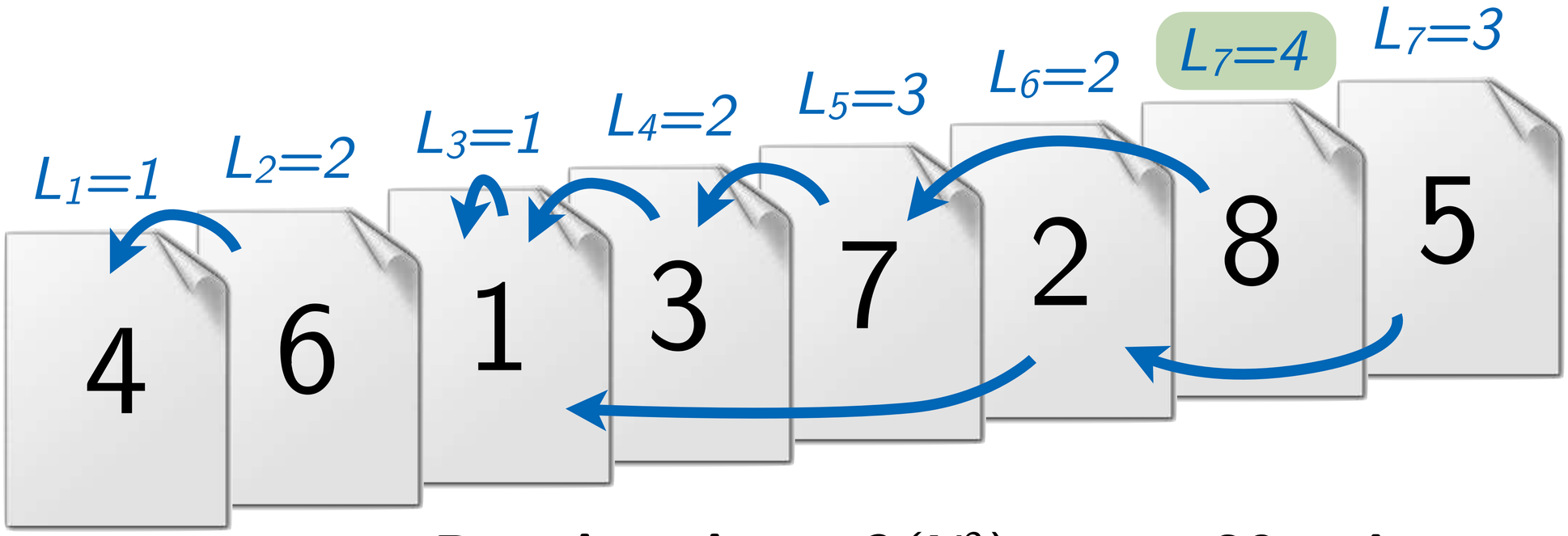

Running time: $O(N^2)$ **30 points**

Subtask 3 / How many longest theses?

Input: N the number of pages

$a_1, a_2, a_3, \ldots, a_N$ the page numbers in the printout

Output: K number of distinct longest dissertations

Limits: $N \leq 5'000$

Subtask 3 / How many longest theses?

Input: N the number of pages

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Output: K number of distinct longest dissertations

Limits: $N \leq 5'000$

Examples		
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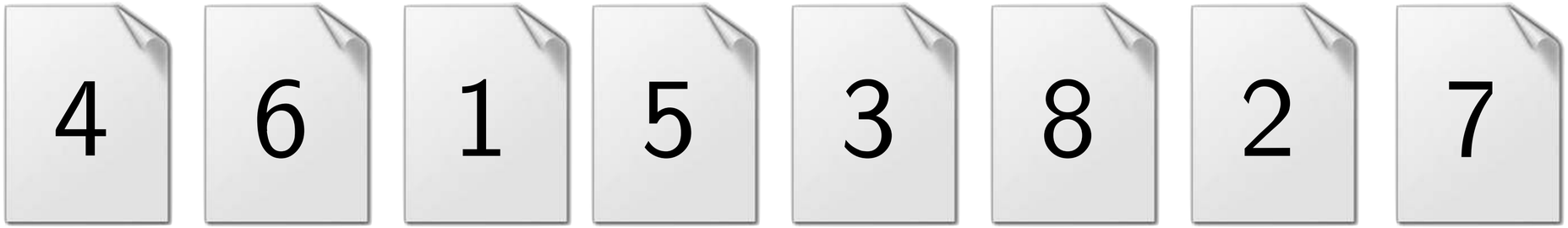
K = 3	K = 4	K = 2

Subtask 3 / How many longest theses?

SOI 2017



Length	
Number	

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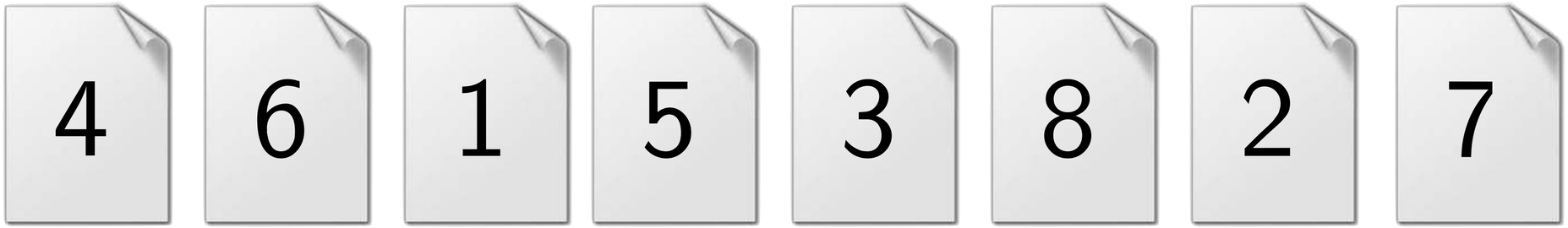

Length	*1*
Number	*1*

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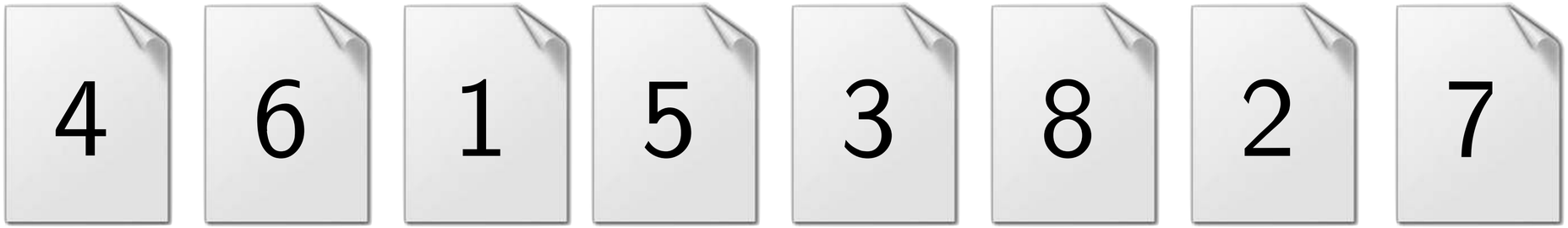
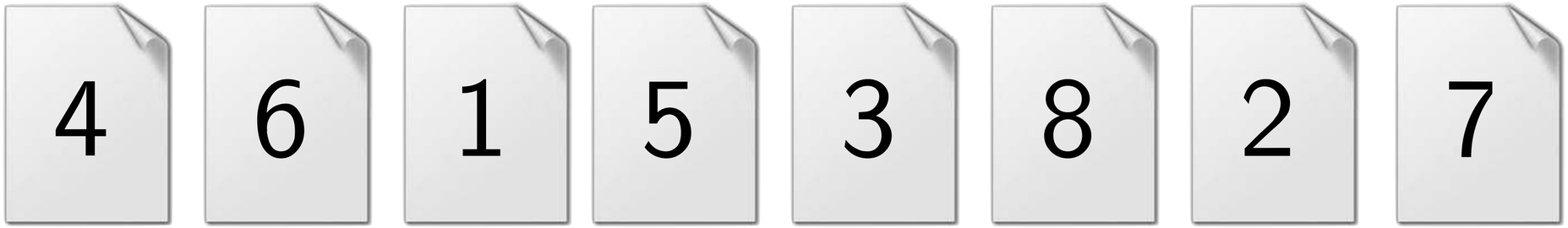


Length	*1*	*2*
Number	*1*	*1*

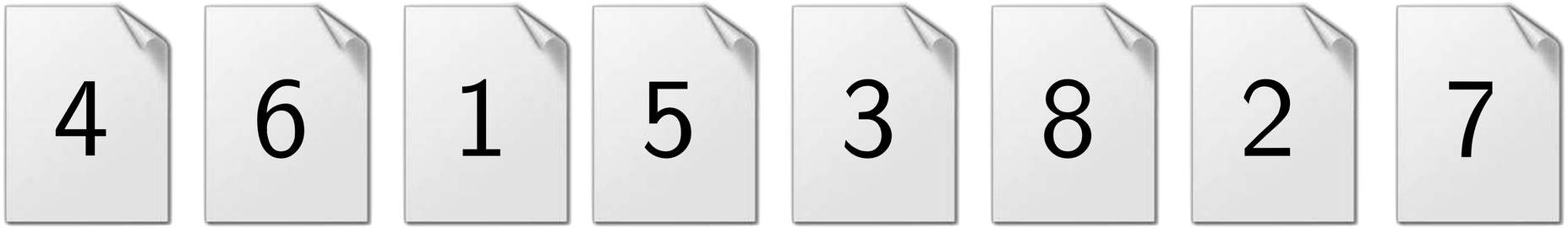
Subtask 3 / How many longest theses?

SOI 2017



Length	*1*	*2*	*1*
Number	*1*	*1*	*1*

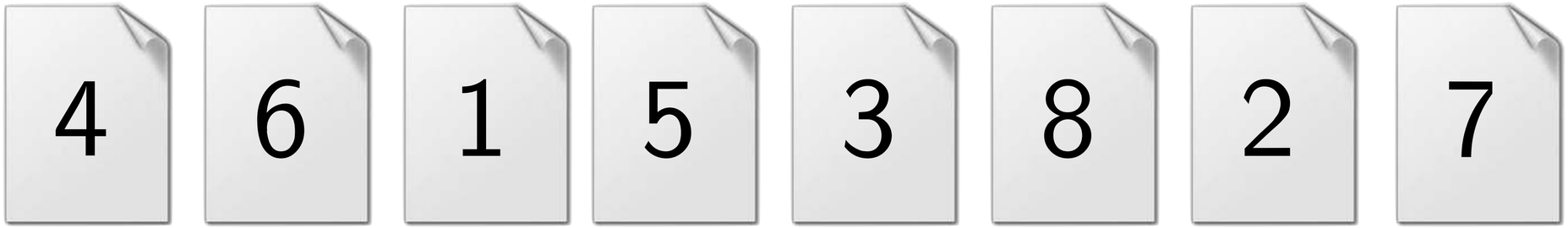
Subtask 3 / How many longest theses?



Length	*1*	*2*	*1*	*2*
Number	*1*	*1*	*1*	*2*

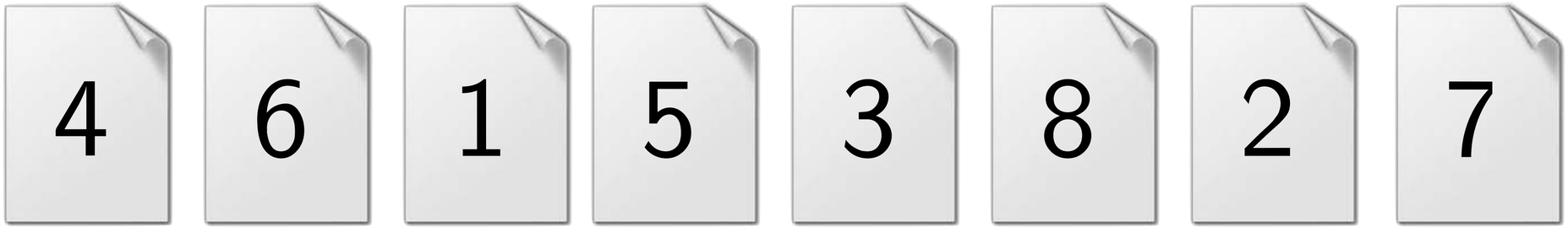
Subtask 3 / How many longest theses?

SOI 2017



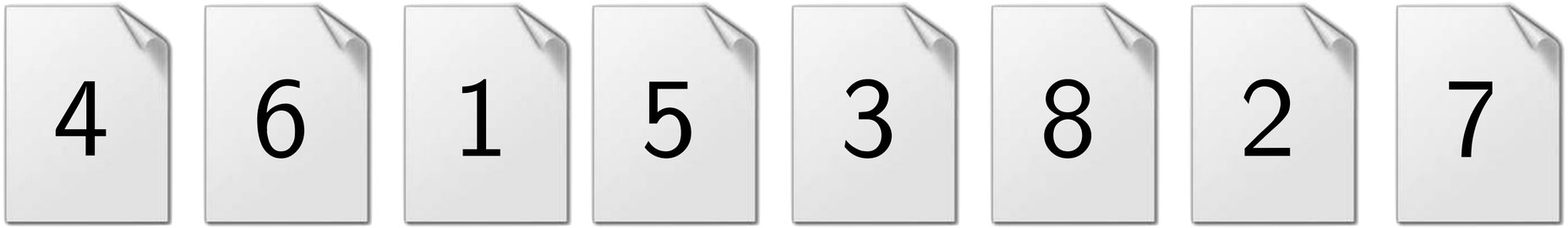
Length	*1*	*2*	*1*	*2*	*2*		
Number	*1*	*1*	*1*	*2*	*1*		

Subtask 3 / How many longest theses?



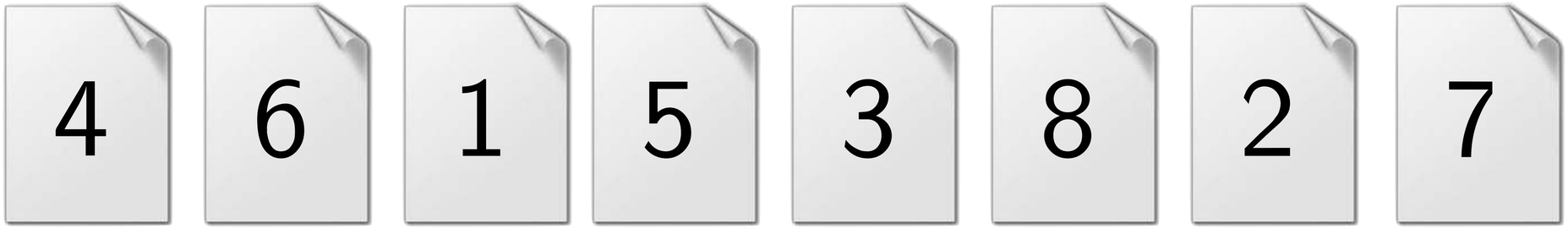
Length	*1*	*2*	*1*	*2*	*2*	*3*	
Number	*1*	*1*	*1*	*2*	*1*	*4*	

Subtask 3 / How many longest theses?



Length	*1*	*2*	*1*	*2*	*2*	*3*	*2*
Number	*1*	*1*	*1*	*2*	*1*	*4*	*1*

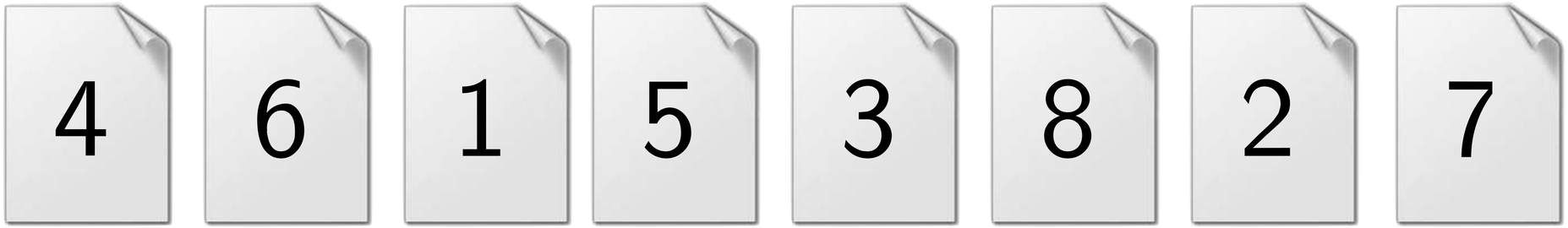
Subtask 3 / How many longest theses?



Length	*1*	*2*	*1*	*2*	*2*	*3*	*2*	*3*
Number	*1*	*1*	*1*	*2*	*1*	*4*	*1*	*5*

Subtask 3 / How many longest theses?

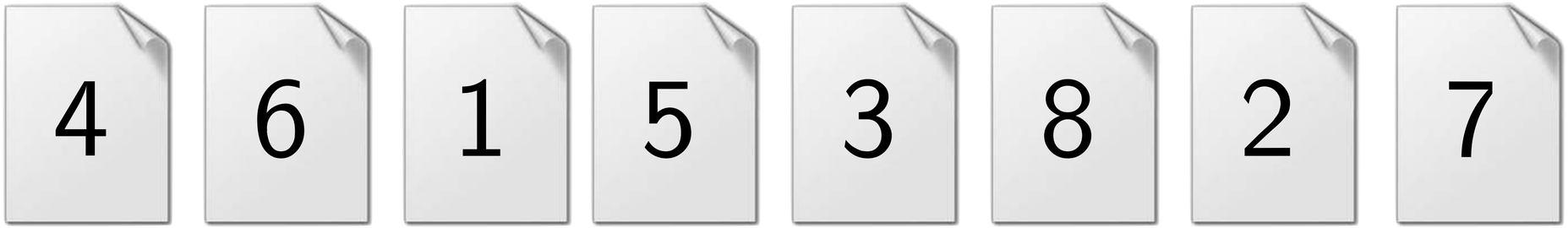
SOI 2017



Length	*1*	*2*	*1*	*2*	*2*	*3*	*2*	*3*
Number	*1*	*1*	*1*	*2*	*1*	*4*	*1*	*5*

$K = 4 + 5 = 9$

Subtask 3 / How many longest theses?



Length	*1*	*2*	*1*	*2*	*2*	*3*	*2*	*3*
Number	*1*	*1*	*1*	*2*	*1*	*4*	*1*	*5*

$K = 4 + 5 = 9$

Running time: $O(N^2)$

50 points

Input: N the number of pages

K number of distinct longest dissertations

Output: $a_1, a_2, a_3, \ldots, a_N$ a thesis containing K longest dissertations

Limits: $200 \leq N \leq 1'000$ and $1 \leq K \leq 10^{18}$

Subtask 4 Reconstruction

Input: N the number of pages

K number of distinct longest dissertations

Output: $a_1, a_2, a_3, \ldots, a_N$ a thesis containing K longest dissertations

Limits: $200 \leq N \leq 1'000$ and $1 \leq K \leq 10^{18}$

Examples

N = 4, K = 4

N = 5, K = 6

N = 200, K = 757·839

Subtask 4 — Reconstruction

Input: N the number of pages

K number of distinct longest dissertations

Output: $a_1, a_2, a_3, \ldots, a_N$ a thesis containing K longest dissertations

Limits: $200 \leq N \leq 1'000$ and $1 \leq K \leq 10^{18}$

Examples

N = 4, K = 4	N = 5, K = 6	N = 200, K = 757·839
4 3 2 1		

Subtask 4 / Reconstruction

Input: N the number of pages

K number of distinct longest dissertations

Output: $a_1, a_2, a_3, \ldots, a_N$ a thesis containing K longest dissertations

Limits: $200 \leq N \leq 1'000$ and $1 \leq K \leq 10^{18}$

Examples

N = 4, K = 4	N = 5, K = 6	N = 200, K = 757·839
4 3 2 1	3 2 1 5 4	

Subtask 4 — Reconstruction

Input: N the number of pages

K number of distinct longest dissertations

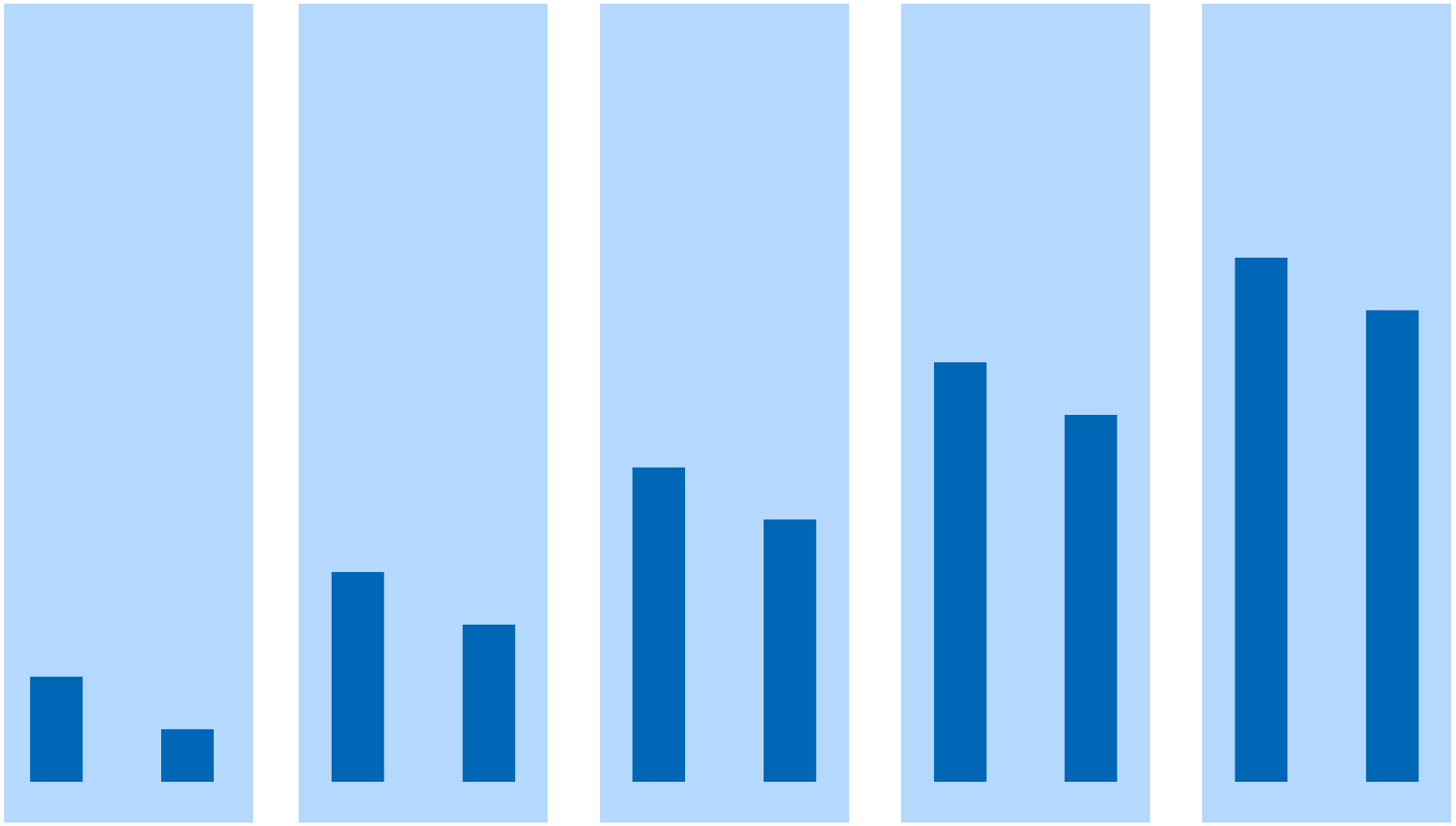
Output: $a_1, a_2, a_3, \ldots, a_N$ a thesis containing K longest dissertations

Limits: $200 \le N \le 1'000$ and $1 \le K \le 10^{18}$

Examples

N = 4, K = 4	N = 5, K = 6	N = 200, K = 757·839
4 3 2 1	3 2 1 5 4	???

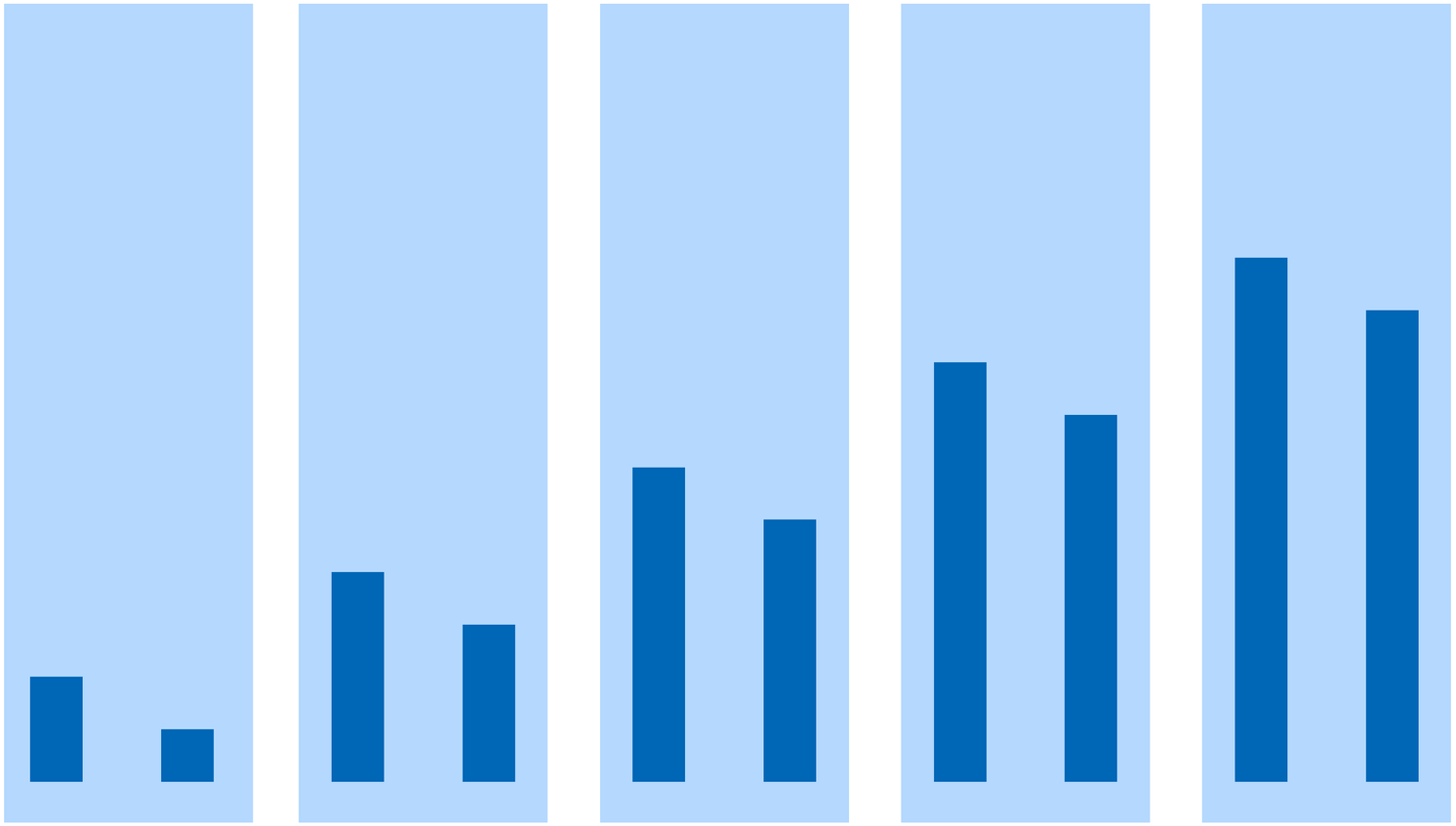
Idea 1: $2^5=32$ options with only $2 \cdot 5=10$ pages



N = 10

M = 5

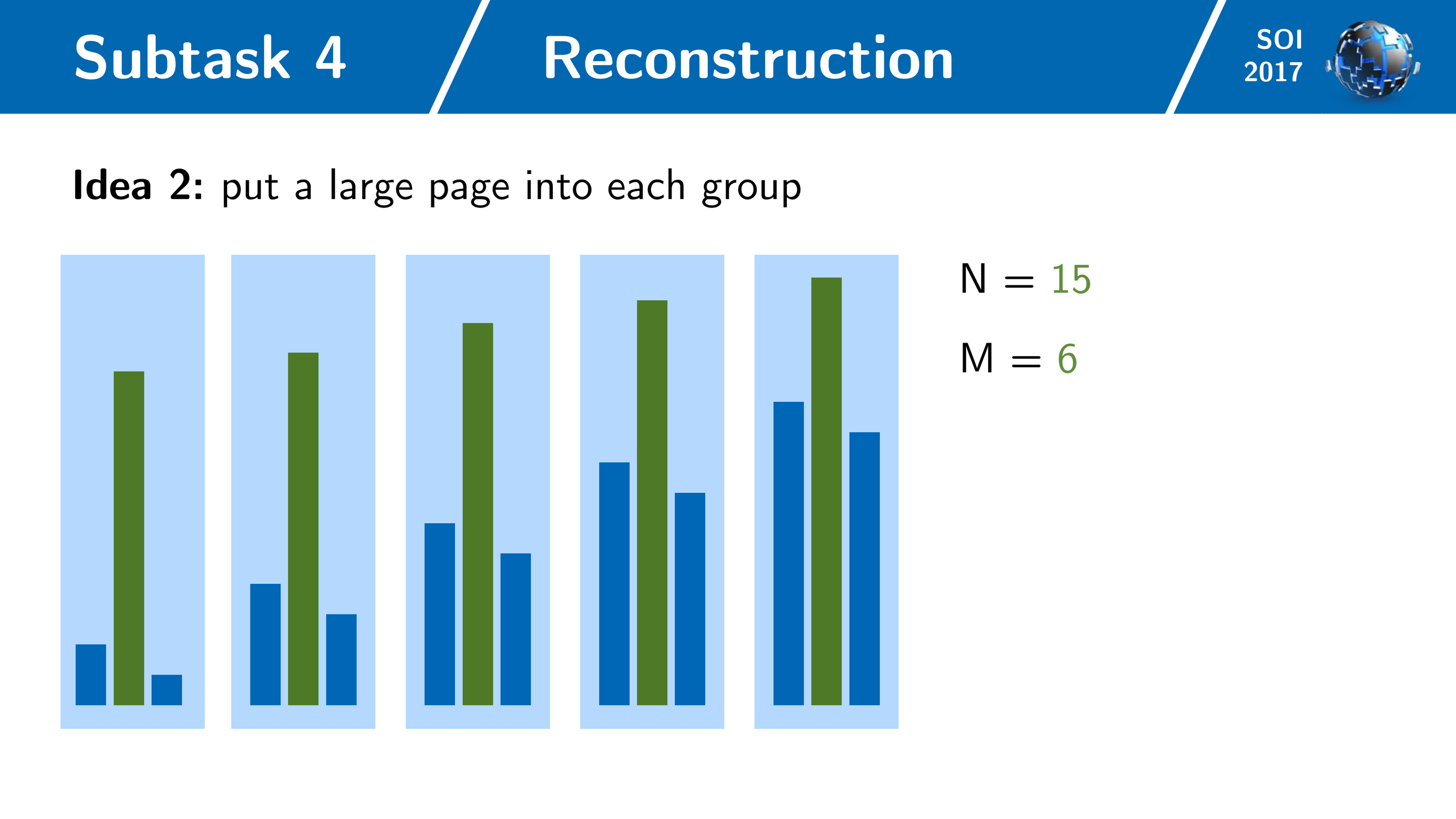
Idea 1: $2^5=32$ options with only $2 \cdot 5=10$ pages



N = 10

M = 5

K = 2 * 2 * 2 * 2 * 2 = 32

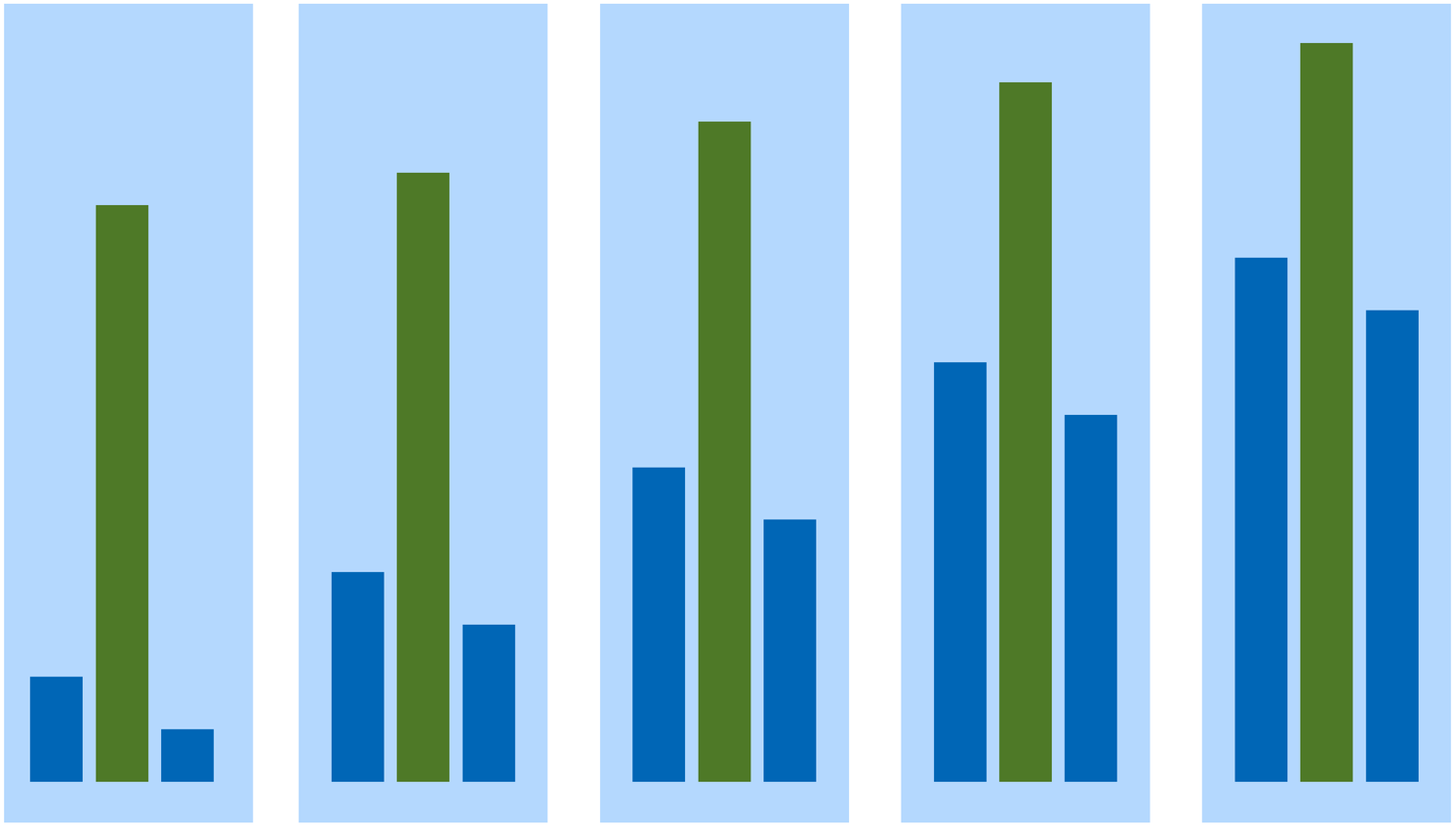


Idea 2: put a large page into each group

$N = 15$

$M = 6$

Idea 2: put a large page into each group

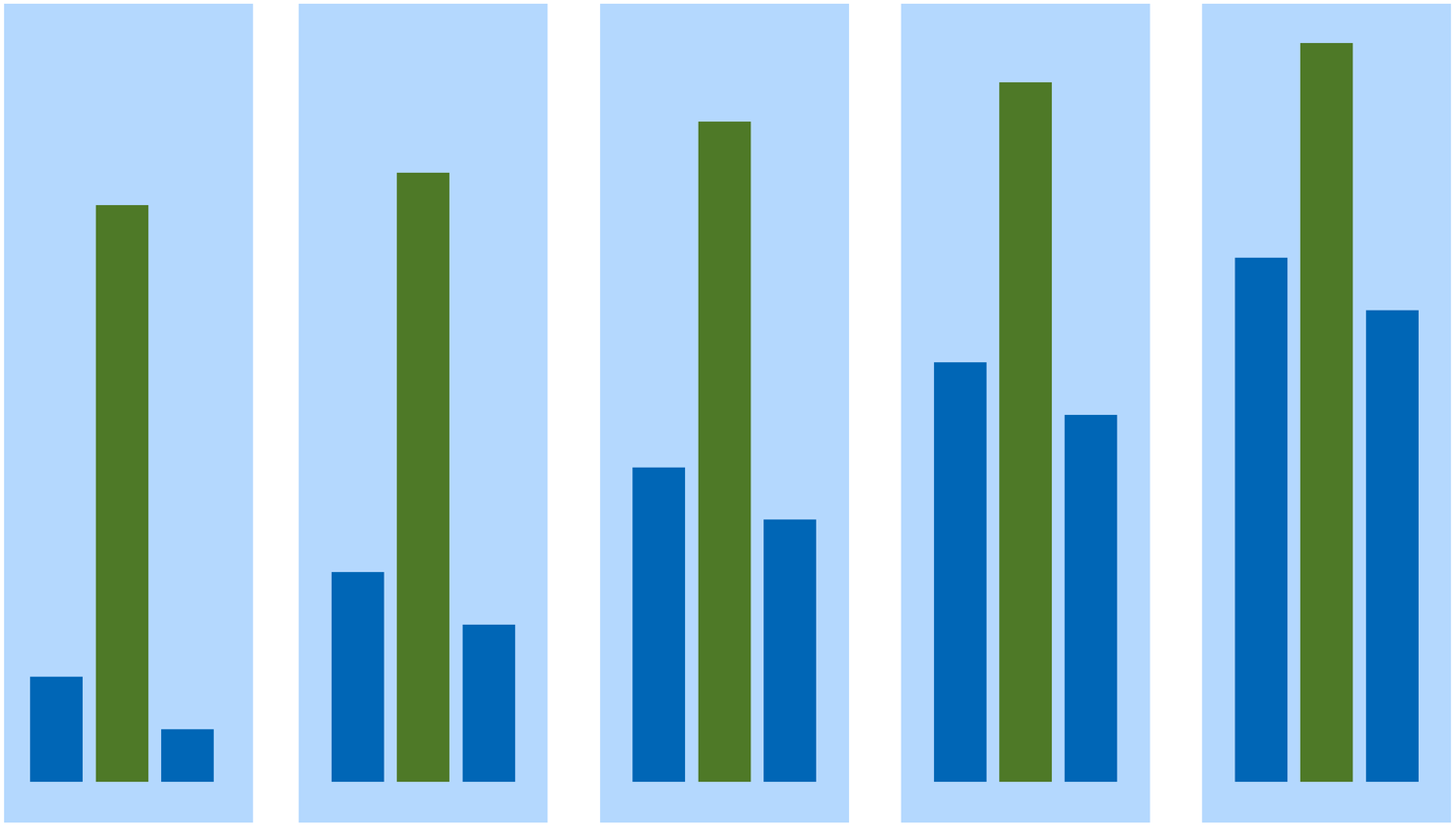


N = 15

M = 6

What is the first green page?

Idea 2: put a large page into each group



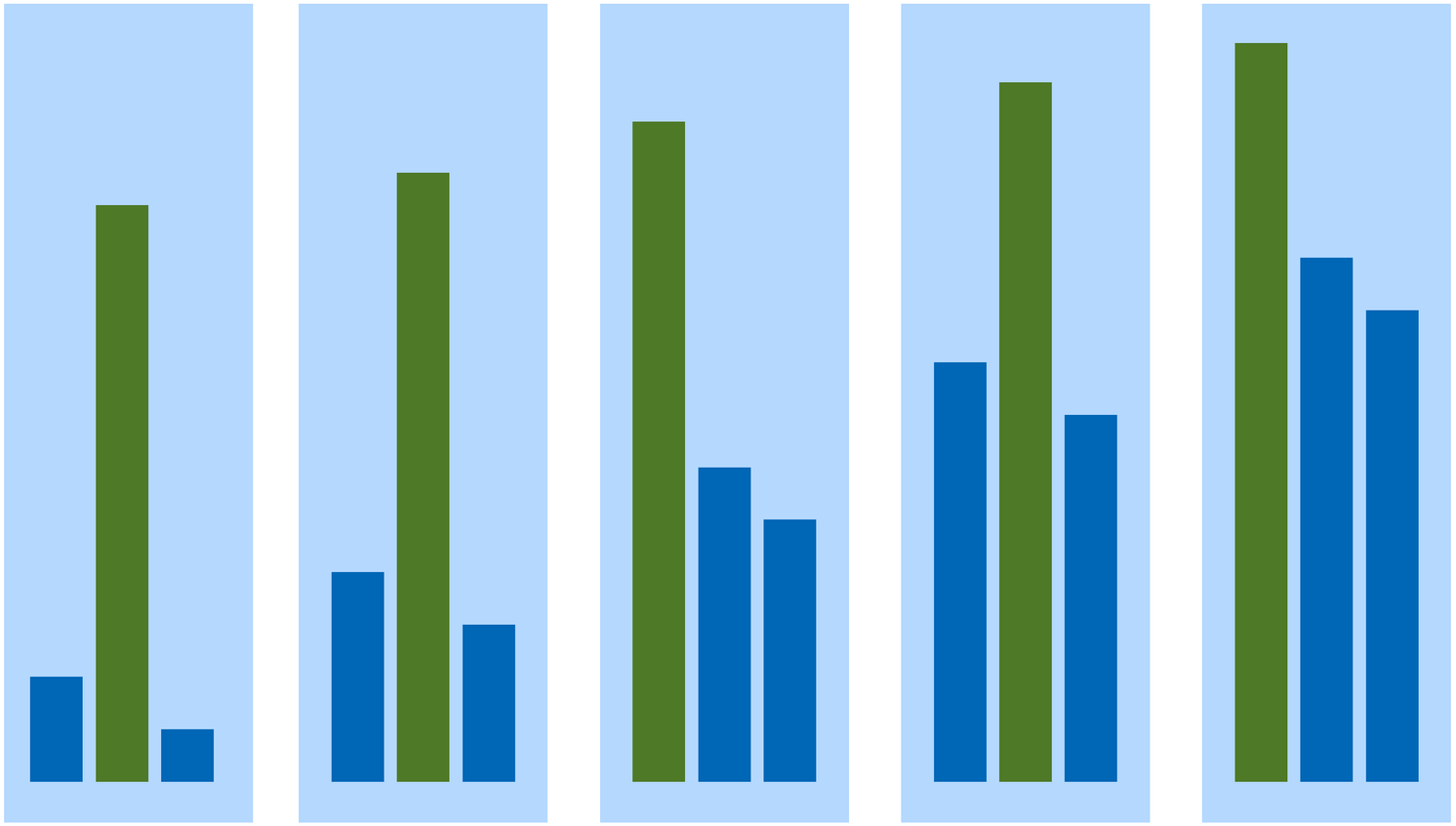
$N = 15$

$M = 6$

What is the first green page?

$K = 1 + 2 + 4 + 8 + 16 = 31$

Idea 3: use green page as on/off switch for this 2^i



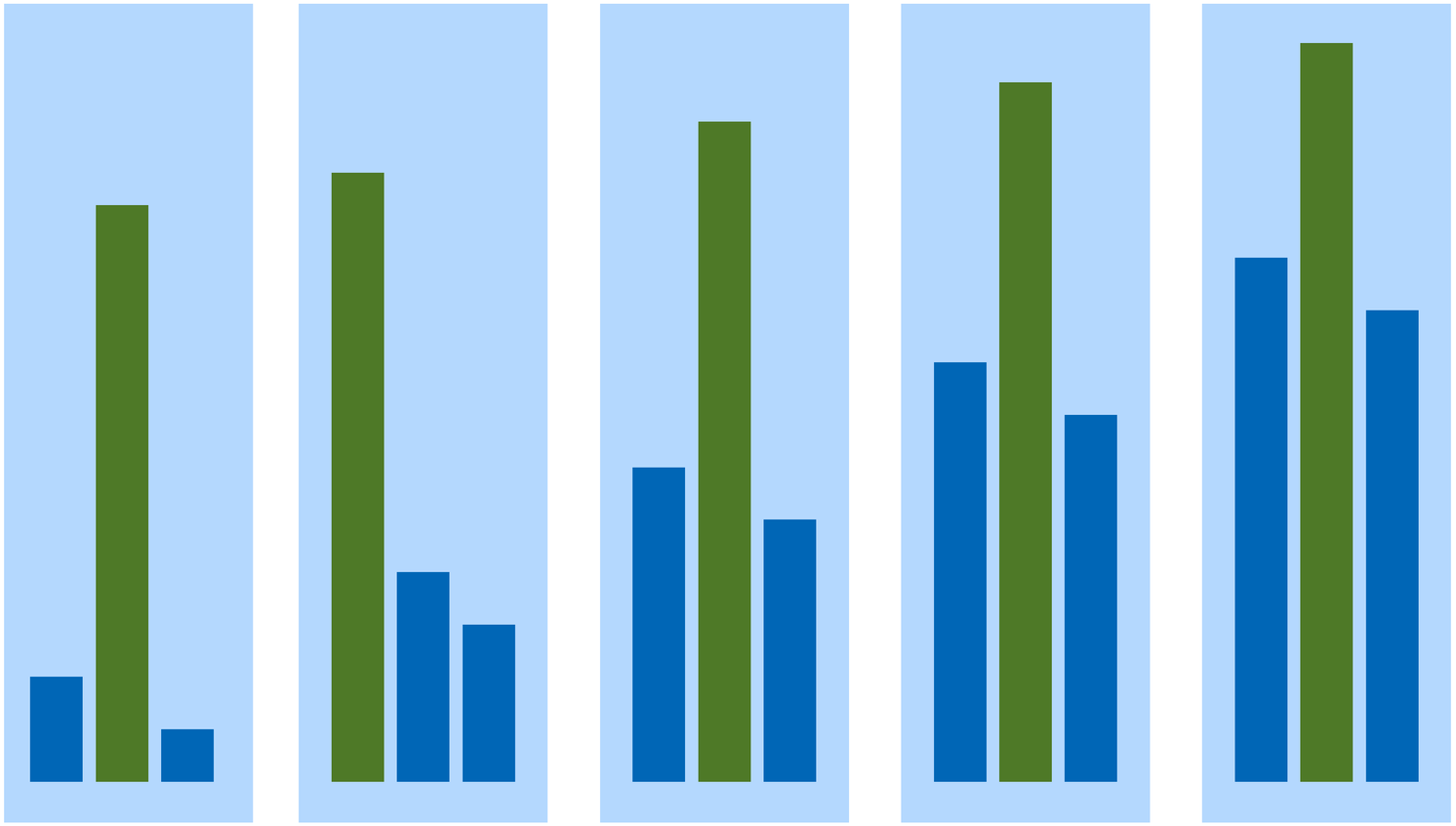
$N = 15$

$M = 6$

What is the first green page?

$K = 1 + 2 + 0 + 8 + 0 = 11$

Idea 3: use green page as on/off switch for this 2^i



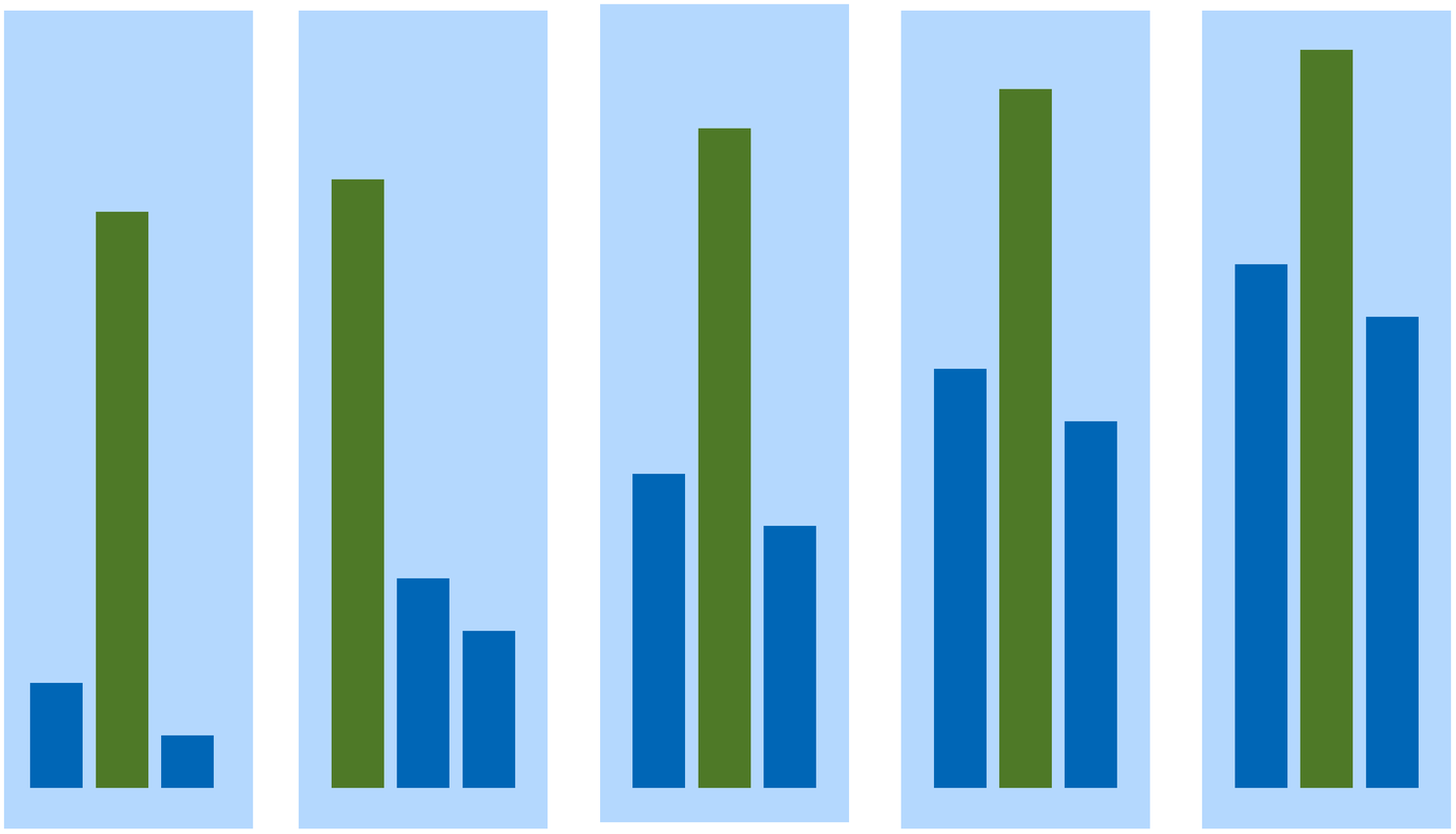
$N = 15$

$M = 6$

What is the first green page?

$K = 1 + 0 + 4 + 8 + 16 = 29$

Subtask 5 / Reconstruction

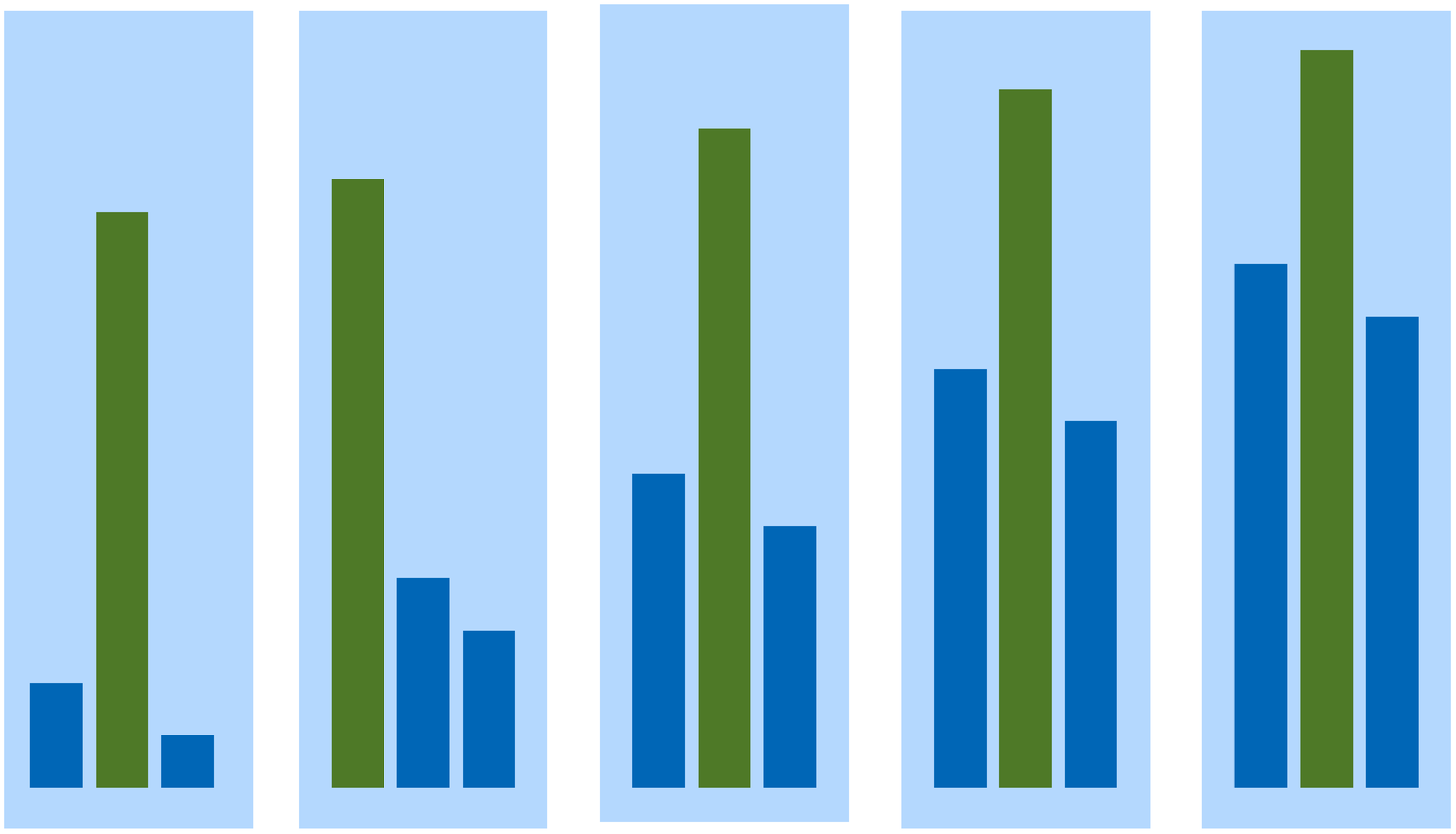


Analysis: We need $3\lceil \log_2(K) \rceil$ pages.

N is always big enough

$3\lceil \log_2(10^{18}) \rceil = 180 < 200$

Subtask 5 / Reconstruction



Analysis: We need $3\lceil \log_2(K) \rceil$ pages.

N is always big enough

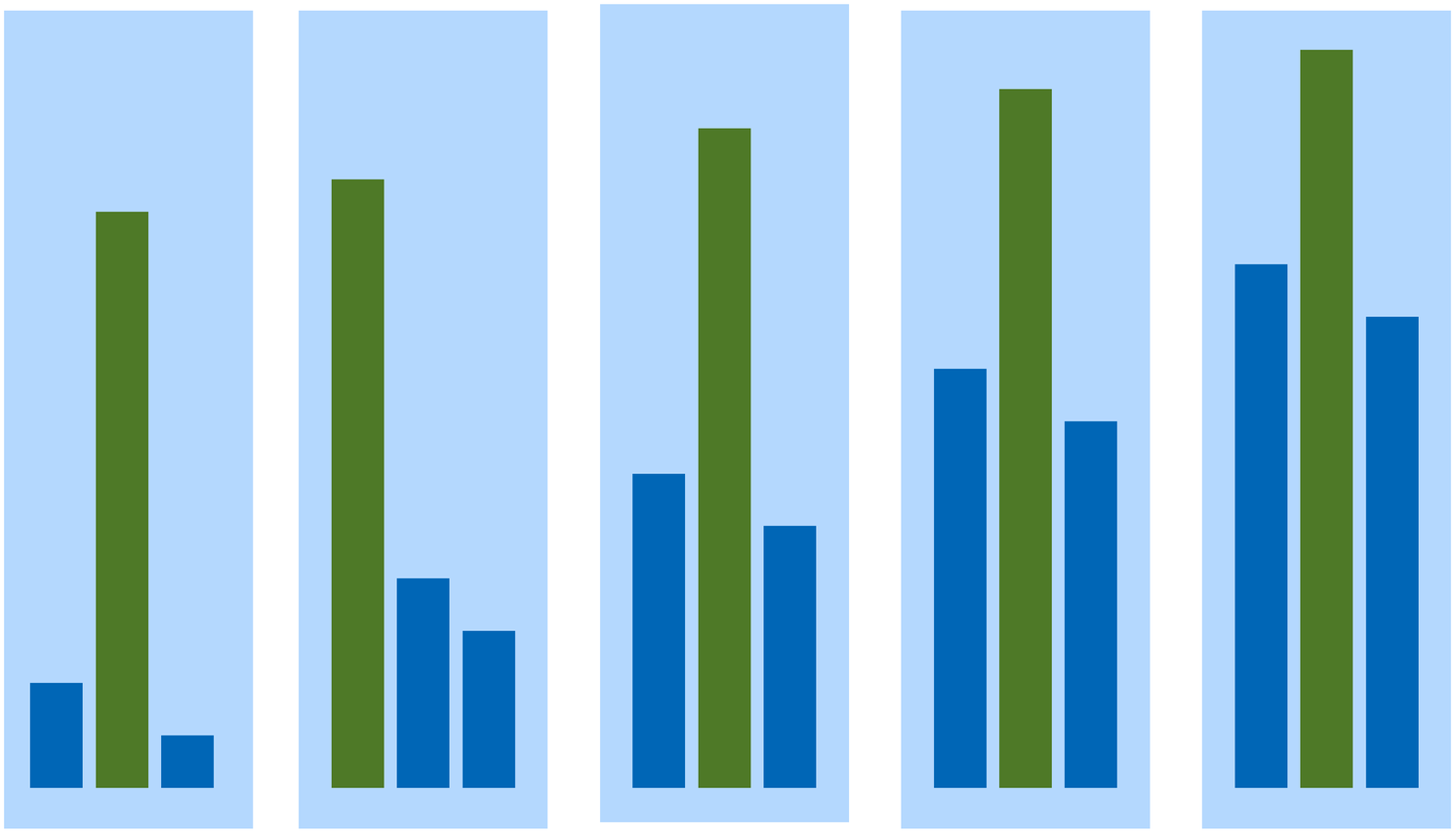
$3\lceil \log_2(10^{18}) \rceil = 180 < 200$

Implementation: **70 points**

Explanation: **97 points**

Subtask 5 / Reconstruction

Analysis: We need $3\lceil \log_2(K) \rceil$ pages.



N is always big enough

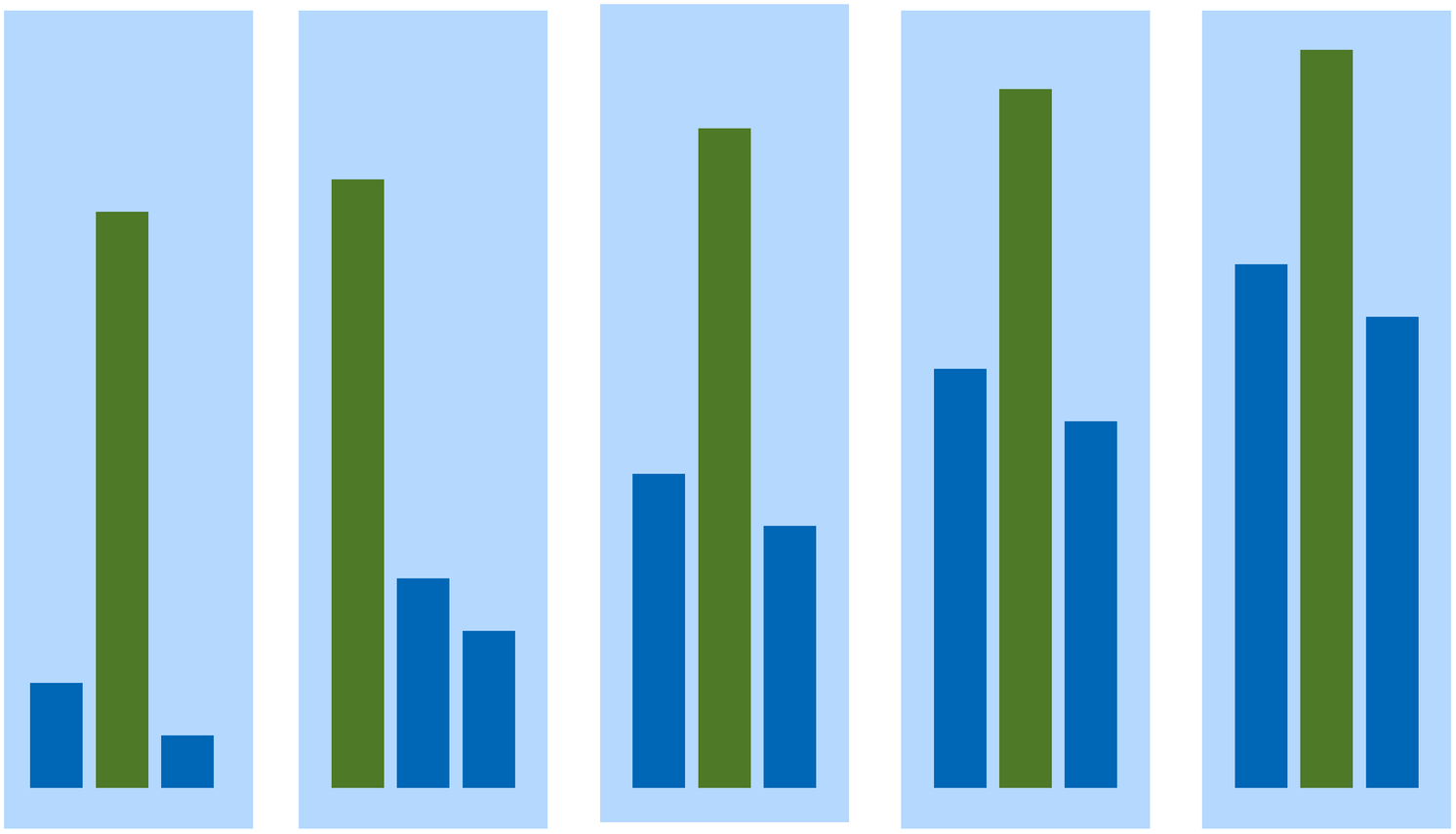
$3\lceil \log_2(10^{18}) \rceil = 180 < 200$

Implementation: **70 points**

Explanation: **97 points**

Improvement: **100 points**

Subtask 5 / Reconstruction



Analysis: We need $3\lceil \log_2(K) \rceil$ pages.

N is always big enough

$3\lceil \log_2(10^{18}) \rceil = 180 < 200$

Implementation: **70 points**

Explanation: **97 points**

Improvement: **100 points**

Groups of size 5:

$2.5\lceil \log_2(K) \rceil$

Best possible?