Numberriddle

SOI Round 1

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Story

Problem: Insert "+" and/or "–" between numbers to reach a given result.

Example: 3 ? 7 ? 6 = 4

Problem: Insert "+" and/or "–" between numbers to reach a given result.

Example: 3 ⑦ 7 ⑦ 6 = 4

Solution: 3 + 7 + 6 = 16

Problem: Insert "+" and/or "–" between numbers to reach a given result.

Example: 3 ⑦ 7 ⑦ 6 = 4

Solution: 3 – 7 + 6 = 2

Problem: Insert "+" and/or "-" between numbers to reach a given result.

Example: 3 ⓘ 7 ⓘ 6 = 4

Solution: 3 + 7 – 6 = 4

Subtask 1: Warm Up

The input consists of only two numbers.

Input: 2 R

a_1 a_2

Output: "YES" or "NO"

Check whether

- $a_1 + a_2 = R$ or
- $a_1 - a_2 = R$.

Subtask 2: One Subtraction

Many numbers, but at most one "-" allowed.

Input: N R

$a_1\ a_2\ a_3\ ...a_N$

Output: "YES" or "NO"

Check whether

- $a_1 + a_2 + a_3 + a_4 + \cdots + a_{n-1} + a_n = R$ or
- $a_1 - a_2 + a_3 + a_4 + \cdots + a_{n-1} + a_n = R$ or
- $a_1 + a_2 - a_3 + a_4 + \cdots + a_{n-1} + a_n = R$ or
- $a_1 + a_2 + a_3 - a_4 + \cdots + a_{n-1} + a_n = R$ or
- $\ldots$
- $a_1 + a_2 + a_3 + a_4 + \ldots - a_{n-1} + a_n = R$ or
- $a_1 + a_2 + a_3 + a_4 + \cdots + a_{n-1} - a_n = R.$

Check whether

- $a_1 + a_2 + a_3 + a_4 + \cdots + a_{n-1} + a_n = R$ or
- $a_1 - a_2 + a_3 + a_4 + \cdots + a_{n-1} + a_n = R$ or
- $a_1 + a_2 - a_3 + a_4 + \cdots + a_{n-1} + a_n = R$ or
- $a_1 + a_2 + a_3 - a_4 + \cdots + a_{n-1} + a_n = R$ or
- $\ldots$
- $a_1 + a_2 + a_3 + a_4 + \ldots - a_{n-1} + a_n = R$ or
- $a_1 + a_2 + a_3 + a_4 + \cdots + a_{n-1} - a_n = R$.

In total $\mathcal{O}(n^2)$ Operations!

Let $S = a_1 + a_2 + a_3 + a_4 + \cdots + a_{n-1} + a_n$.

- $a_1 + a_2 + a_3 + a_4 + \cdots + a_{n-1} + a_n$
- $a_1 - a_2 + a_3 + a_4 + \cdots + a_{n-1} + a_n$
- $a_1 + a_2 - a_3 + a_4 + \cdots + a_{n-1} + a_n$
- $a_1 + a_2 + a_3 - a_4 + \cdots + a_{n-1} + a_n$
- $\ldots$
- $a_1 + a_2 + a_3 + a_4 + \ldots - a_{n-1} + a_n$
- $a_1 + a_2 + a_3 + a_4 + \cdots + a_{n-1} - a_n$

Let $S = a_1 + a_2 + a_3 + a_4 + \cdots + a_{n-1} + a_n$.

- $a_1 + a_2 + a_3 + a_4 + \cdots + a_{n-1} + a_n = S$
- $a_1 - a_2 + a_3 + a_4 + \cdots + a_{n-1} + a_n$
- $a_1 + a_2 - a_3 + a_4 + \cdots + a_{n-1} + a_n$
- $a_1 + a_2 + a_3 - a_4 + \cdots + a_{n-1} + a_n$
- $\ldots$
- $a_1 + a_2 + a_3 + a_4 + \ldots - a_{n-1} + a_n$
- $a_1 + a_2 + a_3 + a_4 + \cdots + a_{n-1} - a_n$

Let $S = a_1 + a_2 + a_3 + a_4 + \cdots + a_{n-1} + a_n$.

- $a_1 + a_2 + a_3 + a_4 + \cdots + a_{n-1} + a_n = S$
- $a_1 - a_2 + a_3 + a_4 + \cdots + a_{n-1} + a_n = S - 2a_2$
- $a_1 + a_2 - a_3 + a_4 + \cdots + a_{n-1} + a_n = S - 2a_3$
- $a_1 + a_2 + a_3 - a_4 + \cdots + a_{n-1} + a_n = S - 2a_4$
- $\ldots$
- $a_1 + a_2 + a_3 + a_4 + \ldots - a_{n-1} + a_n = S - 2a_{n-1}$
- $a_1 + a_2 + a_3 + a_4 + \cdots + a_{n-1} - a_n = S - 2a_n$

Let $S = a_1 + a_2 + a_3 + a_4 + \cdots + a_{n-1} + a_n$ and check whether

- $S = R$ or
- $S - 2a_2 = R$ or
- $S - 2a_3 = R$ or
- $S - 2a_{n-1} = R$ or
- $\cdots$
- $S - 2a_n = R$

Let $S = a_1 + a_2 + a_3 + a_4 + \cdots + a_{n-1} + a_n$ and check whether

- $S = R$ or
- $S - 2a_2 = R$ or
- $S - 2a_3 = R$ or
- $S - 2a_{n-1} = R$ or
- $\cdots$
- $S - 2a_n = R$

In total $\mathcal{O}(n)$ Operations.

Subtask 3: Two Subtractions

Many numbers, but at most two "-" allowed.

Input: N R

a_1 a_2 a_3 ...a_N

Output: "YES" or "NO"

Check whether

- $a_1 + a_2 + a_3 + a_4 + \cdots + a_{n-1} + a_n = R$ or
- $a_1 - a_2 + a_3 + a_4 + \cdots + a_n = R$ or
- $a_1 + a_2 - a_3 + a_4 + \cdots + a_n = R$ or
- $a_1 + a_2 + a_3 - a_4 + \cdots + a_n = R$ or
- ...
- $a_1 + a_2 + a_3 + a_4 + \ldots - a_n = R$ or

- $a_1 - a_2 - a_3 + a_4 + \cdots + a_n = R$ or
- $a_1 - a_2 + a_3 - a_4 + \cdots + a_n = R$ or
- ...
- $a_1 - a_2 + a_3 + a_4 + \ldots - a_n = R$ or
- $a_1 + a_2 - a_3 - a_4 + \cdots + a_n = R$ or
- ...
- $a_1 + a_2 - a_3 + a_4 + \ldots - a_n = R$ or
- $a_1 + a_2 + a_3 - a_4 - \cdots + a_n = R$ or
- ...
- $a_1 + a_2 + a_3 - a_4 + \ldots - a_n = R$ or
- ...
- $a_1 + a_2 + a_3 + a_4 + \ldots - a_{n-2} - a_n = R.$

In total $\mathcal{O}(n^3)$ Operations.

Let $S = a_1 + a_2 + a_3 + a_4 + \cdots + a_{n-1} + a_n$ and check whether

- $S = R$ or
- $S - 2a_2 = R$ or
- $S - 2a_3 = R$ or
- ...
- $S - 2a_n = R$ or
- $S - 2a_1 - 2a_2 = R$ or
- $S - 2a_1 - 2a_3 = R$ or
- ...
- $S - 2a_1 - 2a_n = R$ or
- ...
- $S - 2a_{n-1} - 2a_n = R$.

In total $\mathcal{O}(n^2)$ Operations.

Subtask 4: Countless Possibilities

Many numbers, no restrictions on the number of "-"'s.

Input: N R

$a_1\ a_2\ a_3\ ...a_N$

Output: "YES" or "NO"

Check whether

$$a_1+a_2+a_3+a_4+a_5+a_6+a_7 = R$$
$$a_1+a_2+a_3+a_4+a_5+a_6-a_7 = R$$
$$a_1+a_2+a_3+a_4+a_5-a_6+a_7 = R$$
$$a_1+a_2+a_3+a_4+a_5-a_6-a_7 = R$$
$$a_1+a_2+a_3+a_4-a_5+a_6+a_7 = R$$
$$a_1+a_2+a_3+a_4-a_5+a_6-a_7 = R$$
$$a_1+a_2+a_3+a_4-a_5-a_6+a_7 = R$$
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$$a_1+a_2+a_3-a_4+a_5+a_6+a_7 = R$$
$$a_1+a_2+a_3-a_4+a_5+a_6-a_7 = R$$
$$a_1+a_2+a_3-a_4+a_5-a_6+a_7 = R$$
$$a_1+a_2+a_3-a_4+a_5-a_6-a_7 = R$$
$$a_1+a_2+a_3-a_4-a_5+a_6+a_7 = R$$
$$a_1+a_2+a_3-a_4-a_5+a_6-a_7 = R$$
$$a_1+a_2+a_3-a_4-a_5-a_6+a_7 = R$$
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$$a_1-a_2+a_3+a_4+a_5+a_6+a_7 = R$$
$$a_1-a_2+a_3+a_4+a_5+a_6-a_7 = R$$
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$$a_1-a_2-a_3-a_4-a_5+a_6-a_7 = R$$
$$a_1-a_2-a_3-a_4-a_5-a_6+a_7 = R$$
$$a_1-a_2-a_3-a_4-a_5-a_6-a_7 = R$$

$2^{99} = 633\,825\,300\,114\,114\,700\,748\,351\,602\,688$ possibilities.

A computer would need about 40 000 times the age of the universe to check them.

Check whether

$$a_1+a_2+a_3+a_4+a_5+a_6+a_7 = R$$
$$a_1+a_2+a_3+a_4+a_5+a_6-a_7 = R$$
$$a_1+a_2+a_3+a_4+a_5-a_6+a_7 = R$$
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$$a_1-a_2+a_3+a_4+a_5+a_6+a_7 = R$$
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$$a_1-a_2-a_3-a_4-a_5-a_6+a_7 = R$$
$$a_1-a_2-a_3-a_4-a_5-a_6-a_7 = R$$

$2^{99} = 633\,825\,300\,114\,114\,700\,748\,351\,602\,688$ possibilities.

A computer would need about 40 000 times the age of the universe to check them.

Not possible within the 5 minutes time window on our website!

Input: 3 ⓘ 1 ⓘ 4 ⓘ 1

1. For 3: $\{3\}$

Input: 3 ? 1 ? 4 ? 1

1. For 3: $\{3\}$
2. For 3 ? 1: $\{3 - 1, 3 + 1\} = \{2, 4\}$

Input: 3 ⍰ 1 ⍰ 4 ⍰ 1

1. For 3: $\{3\}$
2. For 3 ⍰ 1: $\{3 - 1, 3 + 1\} = \{2, 4\}$
3. For 3 ⍰ 1 ⍰ 4: $\{2 - 4, 2 + 4, 4 - 4, 4 + 4\} = \{-2, 0, 6, 8\}$

Input: 3 ? 1 ? 4 ? 1

1. For 3: $\{3\}$
2. For 3 ? 1: $\{3 - 1, 3 + 1\} = \{2, 4\}$
3. For 3 ? 1 ? 4: $\{2 - 4, 2 + 4, 4 - 4, 4 + 4\} = \{-2, 0, 6, 8\}$
4. For 3 ? 1 ? 4 ? 1:
   $\{-2 - 1, -2 + 1, 0 - 1, 0 + 1, 6 - 1, 6 + 1, 8 - 1, 8 + 1\} =$
   $\{-3, -1, 1, 5, 7, 9\}$

Input: 3 ⑦ 1 ⑦ 4 ⑦ 1

1. For 3: $\{3\}$
2. For 3 ⑦ 1: $\{3 - 1, 3 + 1\} = \{2, 4\}$
3. For 3 ⑦ 1 ⑦ 4: $\{2 - 4, 2 + 4, 4 - 4, 4 + 4\} = \{-2, 0, 6, 8\}$
4. For 3 ⑦ 1 ⑦ 4 ⑦ 1:
   $\{-2 - 1, -2 + 1, 0 - 1, 0 + 1, 6 - 1, 6 + 1, 8 - 1, 8 + 1\} =$
   $\{-3, -1, 1, 5, 7, 9\}$

a_i's are small (at most 100), possible results are $\leq 100N$.

The list is of size at most $200N$.

In total we have $\mathcal{O}(N^2)$ operations, which is doable in 5mins ($N = 100$).

Questions?