Theoretical Tasks SOI Workshop

Swiss Olympiad in Informatics

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Symmetric Numbers

Mouse Stofl is currently on holidays in the mysterious country of Bitland, where the wise Mouse Brodowskl rules over his empire. Brodowskl has heard of Mouse Stofl's talent to solve problems, but since Mouse Stofl is on holidays and doesn't want to do any work, Stofl needs your help to complete the following task.

In Bitland, palaces are described by a sequence of numbers, such as 23145; the individual digits describe the height of the palace at various positions, if the palace is viewed from the front. On the one hand, mouse Brodowskl loves palaces, but even more so, he loves palaces that are symmetric in appearance, such as the palace described by the number 37973. On the other hand, he also loves to throw parties for all of his people. It is customary in Bitland, to throw a party whenever the (growing) population of Bitland reaches the number corresponding to a palace. Brodowskl is thinking about building another symmetric palace, but he first wants to know by how many people the population has to grow until it matches the palace's number, if the palace is chosen such that this number is minimized.

Task

We say a number is symmetric if its sequence of digits is the same when reading them from left to right as when reading from right to left. So 121, 44544, 7337 are all symmetric, but 233, 9595 and 800 are not symmetric. Note that you should not have leading zeros, so 00800 is not a symmetric number.

Given the population of Bitland N, find the smallest integer K, such that $N + K$ is symmetric.

Input

A single integer N, with $N > 0$

Output

A single integer K, such that $N + K$ is a symmetric number.

Examples

- For $N = 123$, the next symmetric number is $N + K = 131$, therefore $K = 8$.
- For $N = 141$, the next symmetric number is $N + K = 141$, therefore $K = 0$.
- For $N = 3541$, the next symmetric number is $N + K = 3553$, therefore $K = 12$.
- For $N = 129999931$, the next symmetric number is $N + K = 13000031$, therefore $K = 100$.

The Watchtower

You are playing your favorite real-time strategy game. The game is played on a grid in which one is able to move in 8 directions (north, north-east, east, south-east, south, south-west, west and north-west).

You have already built some storehouses. Each storehouse occupies one of the grid cells. Now you need to build a single watchtower which will guard all of them. The watchtower should be built in such a cell that the farthest storehouse will be as close as possible. It is possible but not necessary to build the watchtower on a cell where there is already a warehouse.

In other words, for a given position of a watchtower the *supervision radius* r of the watchtower is the smallest integer such that it is possible to reach any storehouse from the watchtower in at most r steps. Remember that in each step, one is able to move in 8 directions (north, north-east, east, south-east, south, south-west, west and north-west). You want to find the place for the watchtower where r will be as small as possible.

Task

You are given the positions of the storehouses: n distinct points (x_i, y_i).

Find one possible optimal location (x_W, y_W) of the watchtower. If there is more than one optimal solution, output an arbitrary one.

Example

Input: $n = 3$, the points are $\{(1,0), (4,1), (4,4)\}$.
Output: $(2,2)$.

The situation shown below. In the figure, "`.`" represents an empty cell, "`x`" represents a storehouse, and the optimal locations for the watchtower are marked by "`*`". (The coordinates of the cell in the bottom-left corner are $(0,0)$.)

```
....x
.....
..**.
....x
.x...
```

Fingering

Mouse Peter is passionate about playing the piano. He chooses increasingly longer and more demanding pieces to play which he then practices for hours. Of special importance when successfully playing a piece is an impeccable fingering.

For the sake of simplicity we will consider a piece that consist only of one note at a time and that can be played with just the right hand. Only the white keys of the keyboard are used. These white keys are enumerated from left to right from 1 to 50 (7 octaves).

We define a 'fingering' as follows: Beside each note, Peter indicates with which finger he plays that note by writing down a number from the set $\{1, 2, 3, 4, 5\}$.

1. Thumb
2. Index finger
3. Middle Finger
4. Ring finger
5. Little finger

Now, Peter would like to find the most elegant fingering for his piece. A fingering is considered elegant if it can be played with as little movements of the hand as possible. In order to determine a fingering's elegance, he uses following simple method:

A note earns one point of elegance in the following cases:

- The note is the first one of the piece.
- Or the difference between the numbers of the current and the previous note is equal to the difference between the numbers of the current and previous finger. Mathematically: For keys k_i and k_{i+1} fingers f_i and f_{i+1} the following equation must hold: $k_i - k_{i+1} = f_i - f_{i+1}$. Intuitively this means that Peter can place his hand such that there is one finger per key while playing the i-th and $i+1$-th note.

In all other cases the note earns no point of elegance.

Task

Find a fingering which maximizes the points of elegance.

Input

The input consists of two lines. The first line contains an integer N, the number of notes in the piece. The second line contains N integers k_i ($1 \le t_i \le 50$), the keys for notes of the piece.

Output

The output should contain two lines. In the first line, output the optimal number of points of elegance. In the second line, output N numbers f_i ($1 \le f_i \le 5$), separated by spaces, which describe an optimal fingering. If multiple solutions exist, output any of them.

Examples

Example 1 - Hänschen klein (C'=3)[1]

Input	Output
13 7 5 5 6 4 4 3 4 5 6 7 7 7	13 5 3 3 4 2 2 1 2 3 4 5 5 5

Example 2 - Alle meine Entchen (C'=5)

Input	Output
27 5 6 7 8 9 9 10 10 10 10 9 10 10 10 ↪ 10 9 8 8 8 8 7 7 6 6 6 6 5	25 1 2 3 4 5 5 5 5 5 5 4 5 5 5 5 4 3 ↪ 3 3 3 2 2 1 1 1 1 1

Example 3 - Australian National Anthem (C'=1, C"=8)

Input	Output
28 5 8 5 3 5 8 8 8 10 9 8 7 8 9 5 8 5 ↪ 3 1 5 5 6 10 9 8 7 6 5	22 1 4 1 1 3 2 2 2 4 3 2 1 2 3 1 4 1 ↪ 3 1 5 5 1 5 4 3 2 1 1

[1] The notation C'=3 stands for the note number of the middle C. It simply helps you if you want to play the song yourself, but is not relevant for the task itself.

The New Scarf

The winter is coming, hence Mouse Stofl's grandmother decided to make him a beautiful new scarf. She is very modern mouse and therefore she would like to knit him a peace of a modern art: She will use two different knitting patterns that we represent by 0 or 1. Each row of the scarf will contain N of these knitting patterns. Overall, there are 2^N possibilities how these 0s and 1s can be aligned in a row. Grandmother would like to use all of these 2^N different rows in the scarf, each exactly once, to make it very long. Moreover she would like to arrange these 2^N rows in such a way that two consecutive rows will differ in only one place.

The grandmother started to knit but after a few rows she became confused. While knitting a row, she forgot whether she already used the current row or not. Since keeping in mind all the rows used in the scarf is very hard for such an old mouse, she needs your help. Write her a program that will generate a sketch of the scarf so she will know how the scarf should look like.

Task

Write a program that for a given N generates the scarf sketch for Stofl's grandmother. The sketch consist of 0s and 1s that are aligned in overall 2^N rows that each have length N. Each row has to be different and the two consecutive rows have to differ in exactly one 0 or 1. If there are many possible solutions, you can output any of them.

Example

For $N = 3$, a possible output would be:

```
100
101
001
011
111
110
010
000
```